РТУ и С

1) И.С Гоноровский

РТУ и С

2005
1986

2) Баскаков

РТУ и С

1998
2006

3) У.Т. Сиберт

Цепи. Сигналы. Системы.

1988

4) Баев, Кузнецов

Спектральный и Временной Анализ импульсных и переодич. сигналов

Москва МАИ

•) Лекции - Четв. 203^{6}
11-12 зан.

•) Практика нет.

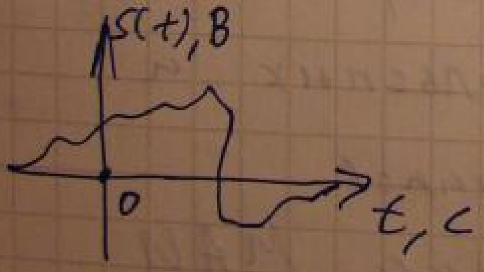
•) Курсовая работа

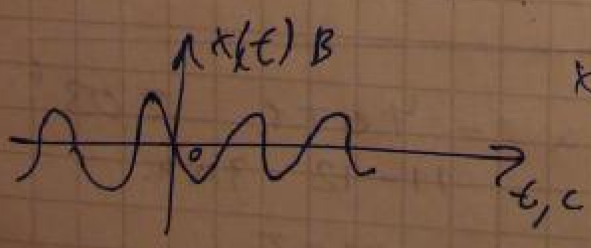
4. Части до 1 дек.

•) Экзамен

1) Спектры сигналов
2) Линейные устр. (цепи)
3) Модуляция и компл. огибающая
4) Нелинейные устройства

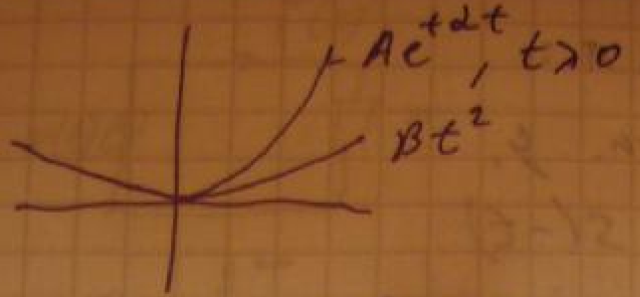
$s(t)$ - сигнал.

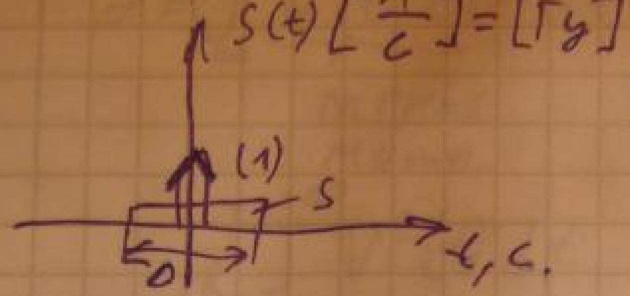




$x(t) = A\cos(\omega t + \varphi)$

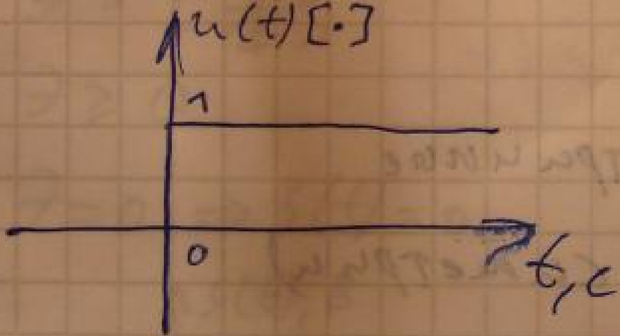
$|s(t)| = C$ - сигнал, огран. по величине.





$$S(t) = \begin{cases} \infty, & t = 0 \\ 0, & t \neq 0 \end{cases}$$

$$\int_{-\infty}^{+\infty} \delta(t)\,dt = 1$$



Переодич. сигнал:

$$s(t) = s(t+T)$$

$s_T(t)$ ⚠ $\qquad \frac{T_0}{T} = N$

Симметрия:

Чётный - сим. отн. y.

$$s(t) = s(-t)$$

even

ev(t)

Нечётный

$$s(t) = -s(-t)$$

odd(t)

$$ev(t) = \cos(\omega t)$$

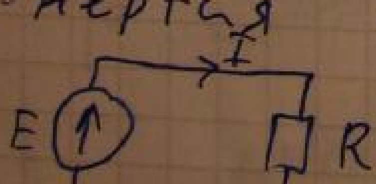
$$odd(t) = \sin(\omega t)$$

Несиметричные

(без симетрии)

<u>Характеристики с.</u>

Энергия сигнала.



$$Э_T = P \cdot T$$

$$P = VI$$

$$Э_T = \frac{V^2}{R} T$$

$$Э = \int_{-\infty}^{+\infty} \underbrace{u(t)\, i(t)}_{\text{мгнов. мощн.}} dt =$$

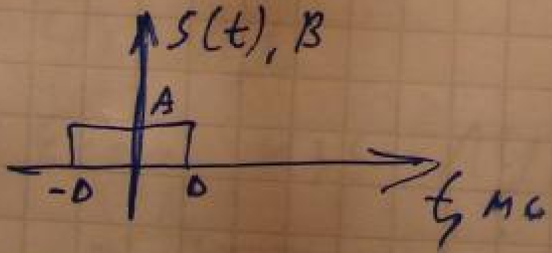
$$= \frac{1}{R} \underbrace{\int_{-\infty}^{+\infty} V^2(t)\, dt}_{\text{энергия сигнала.}} \quad Э_V$$

$$s(t) = Э_s = Э$$

$$Э = \left[\frac{В^2 \cdot c}{4}\right]$$

$$Э \geq 0$$

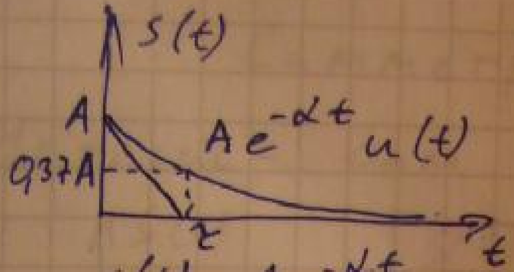
$$Э = 0 \Rightarrow s(t) = 0$$



$A = 3$ В, 4 мс

$$Э = \int_{-Δ}^{Δ} A^2 dt = A^2 t \Big|_{-Δ}^{Δ} = A^2(Δ - (-Δ)) = 2A^2Δ$$

$$Э = 72 \text{ В}^2 \cdot \text{мс}$$



$$s(t) = Ae^{-\alpha t} u(t) = Ae^{-\alpha t}, t>0$$

$$\tau = \frac{1}{\alpha}$$

$$Э = \int_0^{+\infty} A^2 e^{-2\alpha t} dt = A^2 \int_0^{+\infty} e^{-2\alpha t} dt$$

$$= \frac{e^{-2\alpha t}}{-2\alpha}\Big|_0^{+\infty} A^2 = \frac{A^2}{-2\alpha}(0-1) = \frac{A^2}{2\alpha}$$

$\cancel{?}$ – V – В

t – мс, мкс

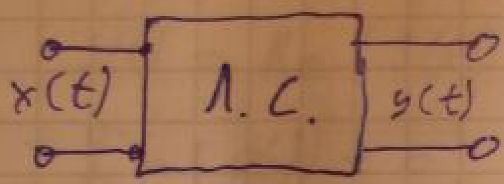
F – кГц, мГц.

Лекция 1. 04.09

Спектр, Преобразование Фурье

(Введение)

1. Частотная характеристика линейной системы



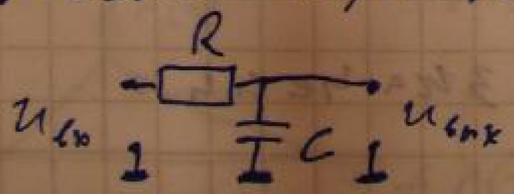
$x(t) = a\cos(2\pi f t + \omega_0)$ $\dot{x} = a e^{j\omega_0}$

$y(t) = A\cos(2\pi f t + W_0)$ $\dot{y} = A e^{jW_0}$

КЧХ $\dot{h} = \frac{\dot{y}}{\dot{x}}$

$h(f)$, при усл. что f - переменная - ЧХ ЛУ.

1) Если изв. ЛУ.



$$u_{вых} = u_{вх} \frac{\frac{1}{j\omega C}}{\frac{1}{j\omega C} + R} = u_{вх} \frac{1}{1 + j\omega RC}$$

$\omega = 2\pi f$

$$H = \frac{1}{1 + j\omega RC}$$

$$H(f) = \frac{1}{1 + j2\pi f RC}$$

2) ЛУ. не известна

(Путём измерения)

$$\frac{A_{вых}}{A_{вх}} = |H(f)| \text{ - АЧХ}$$

$$\omega_{вых} - \omega_{вх} = \arg(H(f)) \text{ - ФЧХ}$$

2. Сигналы

$s(t) = [B]$

$x(t)$ - комп. фу-я

$s(t) = x(t) + jy(t)$ $x(t), y(t)$ - действ.

$$\left.\begin{array}{l} \xrightarrow{x(t)} \\ \xrightarrow{y(t)} \end{array}\right\} \xrightarrow{s(t)}$$

Свойства:

1. Периодичность П/Н
2. Симметрия Четн/Нечётн/Никакая
   even odd
3. Ограничен. по значению
   $|s(t)| < C$
4. Абсолютная интегрируемость
   $$\int_{-\infty}^{+\infty} |s(t)|\, dt \le M$$
   - сигнал абс. интегрируем.

5. ~~Харак~~ Энергия с.

$$Э = \int_{-\infty}^{+\infty} |(S(t)|)^2 \, dt = [B^2 \cdot c]$$

Э – действительная, ≥ 0

$\boxed{Э \leq Э_0}$ – сигнал с конечной энергией

$S(t)$ – действ.

$s(t) = Re(S(t))$

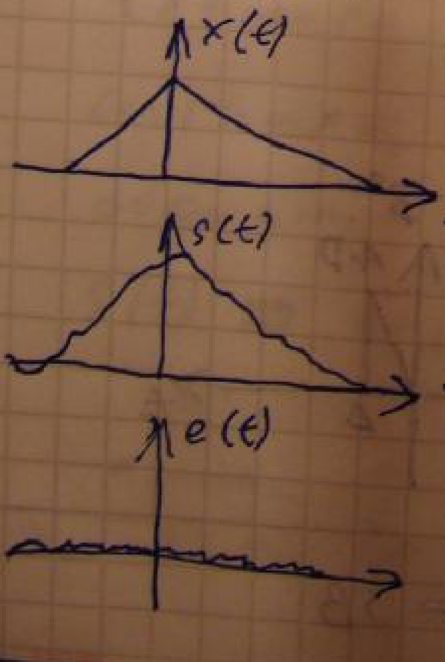
$Im(s(t)) = 0$

$$Э = \int_{-\infty}^{+\infty} s^2 \, dt$$

3. Сравнение сигналов

1) Визуально посмотреть на график
2) Ввести правило сравн., выраж. математически.

$x(t)$ $s(t)$



$|x(t) - s(t)| = e(t)$ – сигнал ошибки

$$Э_e = \int_{-\infty}^{+\infty} |e^2| \, dt =$$

$$= \int_{-\infty}^{+\infty} |x - s|^2 \, dt =$$

$$= \int_{-\infty}^{+\infty} (x(t) - s(t))^2 \, dt$$

$Э_e \geq 0$

$$\underbrace{\int_{-\infty}^{+\infty} x^2(t)dt}_{Э_x>0} - \int_{-\infty}^{+\infty} 2x(t)s(t)dt + \underbrace{\int_{-\infty}^{+\infty} s^2(t)dt}_{Э_s>0}$$

$$Э_e = Э_x + Э_s - 2\underbrace{\int_{-\infty}^{+\infty} x(t)s(t)dt}_{*}$$

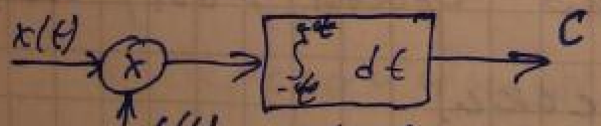
Чем * больше, тем различие меньше.

Чем * меньше, тем различие больше.

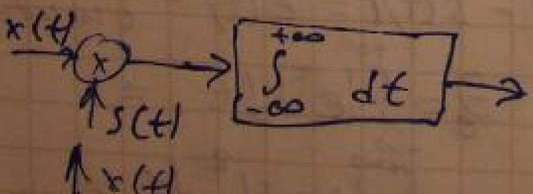
Кореляционный интеграл.

корреляция – особого рода похожесть

3.1 Коррелятор



Δt – время кореляции



Δ = 1 мкс A = 2B D = 3B

$$C = \frac{\Delta \cdot A \cdot D}{2} = \frac{2 \cdot 3 \cdot 1}{2} = 3\ \text{В}^2\text{мкс}$$

$$[C] = [Э]\ !$$

Результат корреляции любого чётного сигнала с любым нечётным всегда равен 0

$ev(t)\, ev(t) = ev(t)$

$odd(t)\, odd(t) = ev(t)$

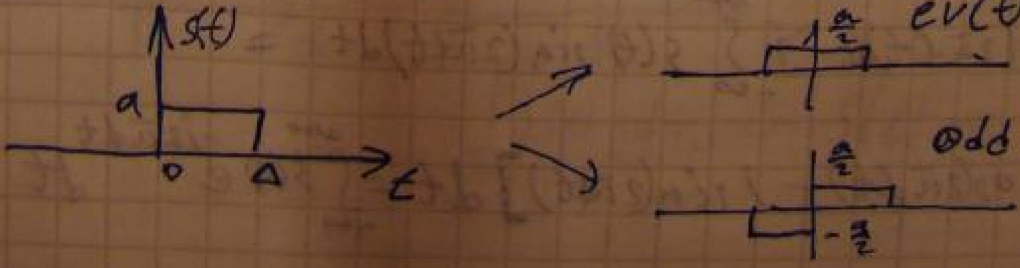
$ev(t)\, odd(t) = odd(t)$

4. Корреляция с гармоникой

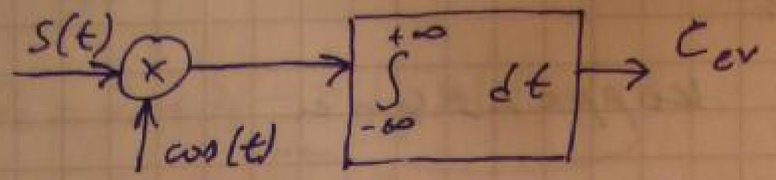
4.1. Теорема

Любой сигнал можно представить суммой чётного и нечётного сигнала, причём единств. образом.

$s(t) \Rightarrow ev(t) + odd(t)$



4.2 Корреляция с косинусом и синусом



$$\int_{-\infty}^{+\infty} \cos(\overset{2\pi f t}{t})\, s(t)\, dt = \int_{-\infty}^{+\infty} odd(t)\cos(2\pi f t) + ev(t)\cos(2\pi$$

$$= \underbrace{\int_{-\infty}^{+\infty} odd(t)\cos(2\pi f t)dt}_{0} + \underbrace{\int_{-\infty}^{+\infty} ev(t)\cos(2\pi f t)}_{C_{ev}} = C_{ev}$$

$$\int_{-\infty}^{+\infty} \sin(2\pi f t)\, dt = \underbrace{\int_{-\infty} odd(t)\sin(2\pi f t)}_{C_{odd}} + \underbrace{\int_{-\infty} ev(t)\sin(2\pi f t)}_{0} =$$

Чтобы объединить рез. корреляции с кос. и син., восп. методом комп. амплитуды (КА)

$$\cos(2\pi f) \Leftrightarrow 1$$

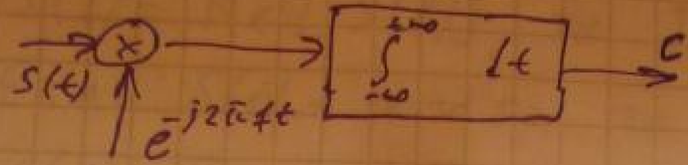
$$\sin(2\pi f) \Leftrightarrow e^{-j\frac{\pi}{2}} \Leftrightarrow -j$$

$$\dot{C} = C_{ev} - j\, C_{odd}$$

$$\int_{-\infty}^{+\infty} s(t)\cdot\cos(2\pi f t)dt - j\int_{-\infty}^{+\infty} s(t)\sin(2\pi f t)dt =$$

$$= \int_{-\infty}^{+\infty} s(t)\left[\cos(2\pi f t) - j\sin(2\pi f t)\right] dt = \int_{-\infty}^{+\infty} s(t)\, e^{-j2\pi f t} dt$$

$$\boxed{e^{ix} = \cos x + i \sin x}$$ Ф. Эйлера



5. Преобразование Фурье

Спектр сигнала

$C(f) = S(f)$ - спектр сигнала $s(t)$

$$S(f) = \int_{-\infty}^{+\infty} s(t)\, e^{-j2\pi f t}\, dt$$

Преобразование Фурье

$$S(f) = \mathcal{F}\left[s(t) \right]$$

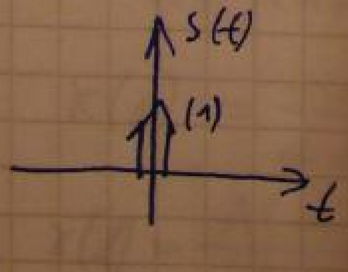
$$s(t) = \int_{-\infty}^{+\infty} S(f)\, e^{j2\pi f t}\, df$$

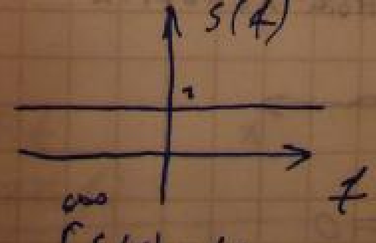
$$s(t) = \mathcal{F}^{-1} S(f)$$

$$S(f) = \left[B \cdot c \right] = \left[\frac{B}{Гц} \right]$$ - спектральная плотность

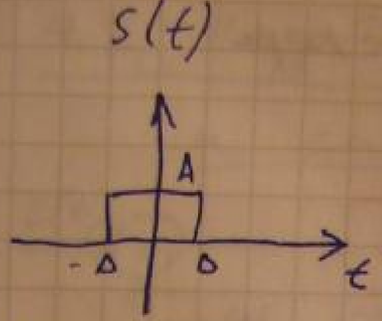
6. Спектры простых сигналов

$\delta(t)$





$\int_{-\infty}^{\infty} \delta(t)\, 1\, dt$ $\qquad S(f)$ для $\delta(t) = const$



Намеренно расположить сигнал чётно (!)

$$s(t)=\begin{cases}A & |t|<\Delta\\ 0 & |t|>\Delta\end{cases} = A, |t|\le\Delta$$

$$S(f)=\int_{-\Delta}^{\Delta} A e^{-j2\pi ft}\,dt = \frac{A}{-j2\pi f} e^{-j2\pi ft}\Big|_{-\Delta}^{\Delta} =$$

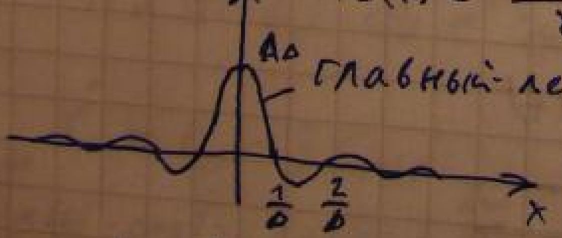
$$= \frac{A}{-j2\pi f}\left(e^{-j\pi f\Delta} - e^{j\pi f\Delta}\right) =$$

$$\boxed{e^{ix}+e^{-ix} = 2\cos x}$$

$$= \frac{A(\cancel{-2j})\sin(\pi f\Delta)}{\cancel{-2j}\pi f} = \frac{A}{\pi f}\sin(\pi f\Delta) =$$

$$= A\Delta\,\mathrm{sinc}(\pi f\Delta)$$

$\mathrm{sinc}(x) = \frac{\sin(x)}{x}$



$\mathrm{sinc}(\pi f\Delta) = 0$

$\sin(\pi f\Delta) = 0$

$\pi f\Delta = \pi\cdot N$, $\pi f\Delta \neq 0$

$N = \pm1\ \pm2 \ldots$

Если $s(f)$ – чётн., то

$s(f) = Re[S(f)]$, т.е. $Im[S(f)] = 0$

Если $S(t)$ – нечётн., то

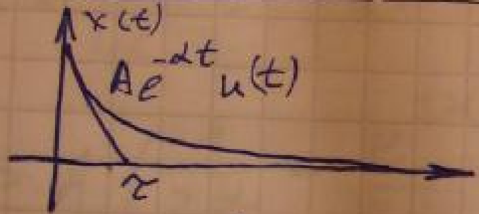
$s(f) = Im[S(f)]$, т.е. $Re[S(f)] = 0$

Свойство площади

$\sqcap\!\sqcup[s(t)] = S(0)$

$\sqcap\!\sqcup[s(f)] = s(0)$

$\boxed{\text{Площадь sinc – площ. треугольн. вписанного в главный лепесток}}$



$$x(t) = Ae^{-\alpha t}, t > 0$$

$$x(f) = \int_{-\infty}^{+\infty} Ae^{-\alpha t} e^{-j2\pi f t} dt =$$

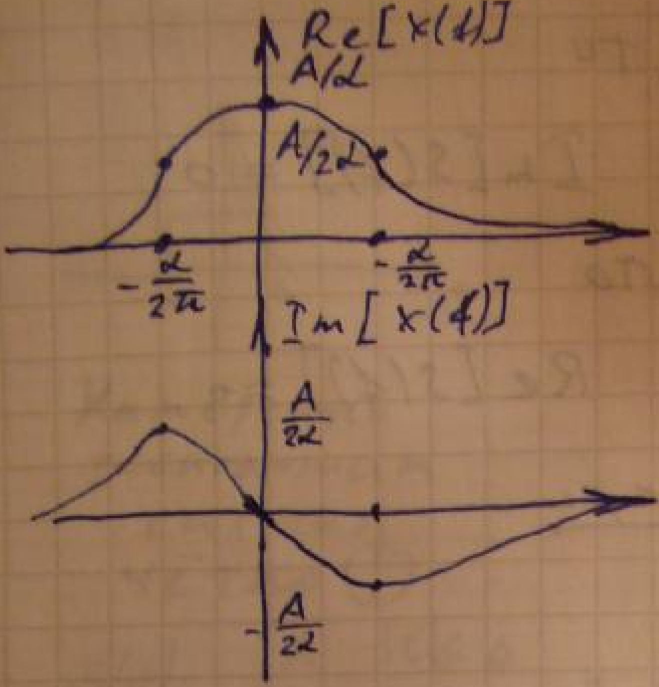
$$= A\int_{0}^{+\infty} e^{-(\alpha + j2\pi f)t} dt = \frac{A}{-(\alpha + j2\pi f)} e^{-(\alpha + j2\pi f t)}\Big|_{0}^{\infty}$$

$$= \frac{A}{\alpha + j2\pi f}(0 - 1) = -\frac{A}{\alpha + j2\pi f}$$

$$x(f) = \frac{A}{\alpha + j2\pi f}$$

$$x(f) = \frac{A(\alpha - j2\pi f)}{(\alpha + j2\pi f)(\alpha - j2\pi f)} =$$

$$= \frac{A\alpha - j2\pi f A}{\alpha^2 - (2\pi f)^2} = \underbrace{\frac{A\alpha}{\alpha^2 + (2\pi f)^2}}_{Re} + j\underbrace{\frac{2\pi f A}{\alpha^2 + (2\pi f)^2}}_{Im}$$



$Re[x(f)] = ev(f)$

$Im[x(f)] = odd(f)$

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Семинар 2 08.09

$$s(t) = \begin{cases} A e^{-\alpha t} & 0 < t < 2/\alpha \\ A e^{\alpha t} & -2/\alpha < t < 0 \end{cases}$$

$$\int_{-\infty}^{+\infty} s^2\,dt = \underbrace{\int_{-2/\alpha}^{0} (A e^{+\alpha t})^2\,dt}_{I} + \int_{0}^{2/\alpha} (A e^{-\alpha t})^2\,dt$$

$$I = \int_{-2/\alpha}^{0} A^2 e^{2\alpha t}\,dt = A^2 \frac{e^{2\alpha t}}{2\alpha}\Bigg|_{-2/\alpha}^{0} =$$

$$= A^2\left(\frac{1}{2\alpha} - \frac{e^{2\alpha\frac{-2}{\alpha}}}{2\alpha}\right) \approx A^2\left(\frac{1}{2\alpha} - \frac{0{,}02}{2\alpha}\right) = \frac{A^2}{2\alpha}\,0{,}98$$

$$II = \int_{0}^{2/\alpha} A^2 e^{-2\alpha t}\,dt = A^2 \frac{e^{-2\alpha t}}{-2\alpha}\Bigg|_{0}^{2/\alpha} = \frac{A^2}{2\alpha}\,0{,}98$$

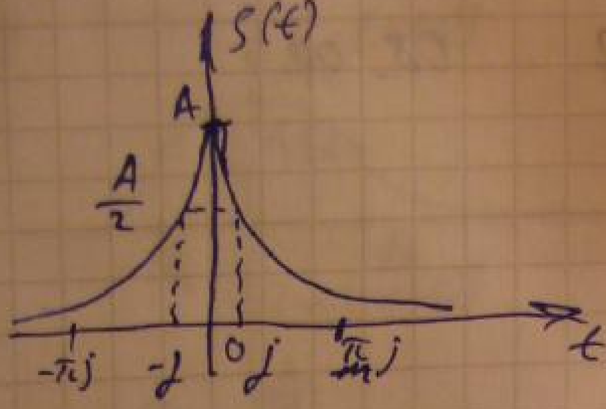
$$Э = 0{,}98\,\frac{A^2}{\alpha}$$

$A = 3$ В

$\Delta = 4$ мс

$\alpha = 0{,}5$ кГц

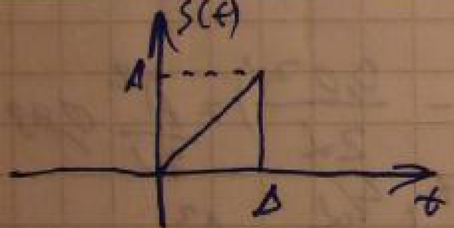
$$Э = 0{,}98\,\frac{9}{0{,}5} = 17{,}6\ \frac{В^2}{кГц}\ \ (В^2 \cdot мс)$$



$$S(t) = A\frac{\gamma^2}{t^2+\gamma^2}$$

$$Э = \int_{-\infty}^{+\infty}\left(A\frac{\gamma^2}{t^2+\gamma^2}\right)^2 dt = A^2\gamma^4\int_{-\infty}^{+\infty}\frac{dt}{(t+j\gamma)^2(t-j\gamma)^2}$$

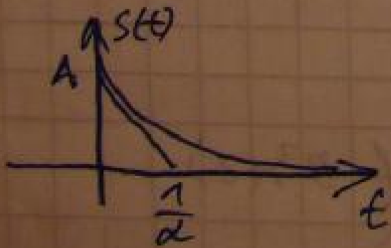
$$= A^2\gamma^4\left[\frac{1}{2\pi j}\,\frac{1}{(2-1)!}\left[\frac{d}{dt}\,\frac{1}{(t-j\gamma)^2}\right]\right]_{t=j\gamma}$$



$S(t) = \frac{A}{\Delta}t$ при $t \in (0;\Delta]$

$$Э = \int_0^{\Delta}\left(\frac{A}{\Delta}t\right)^2 dt = \frac{A^2}{\Delta^2}\left|\frac{t^3}{3}\right|_0^{\Delta} =$$

$$= \frac{A^2}{\Delta^2}\left(\frac{\Delta^3}{3} - 0\right) = \frac{A^2\Delta}{3}$$

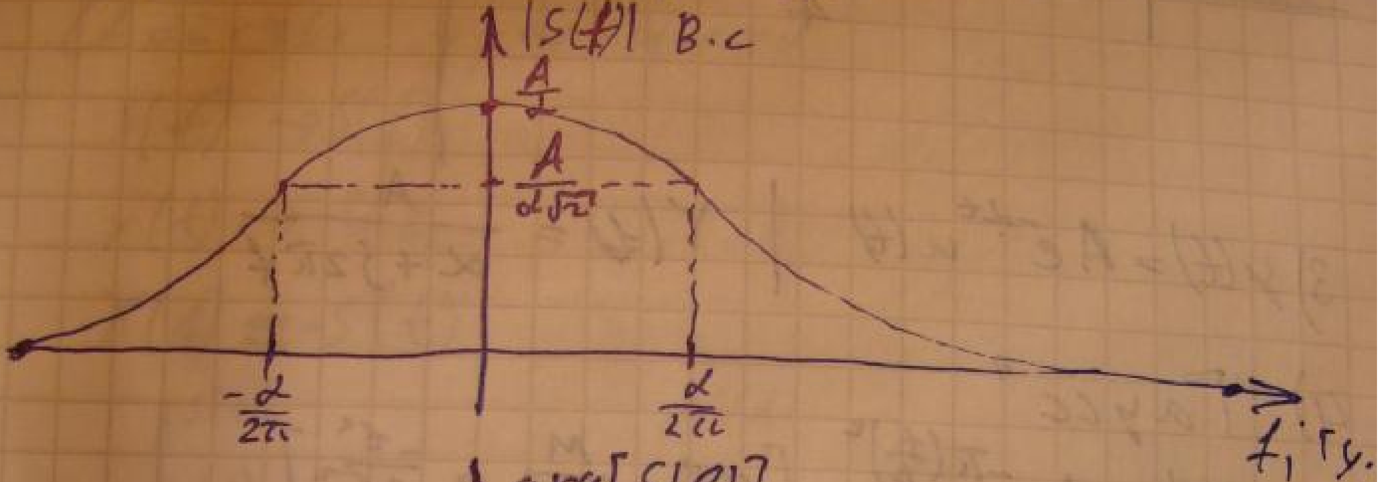


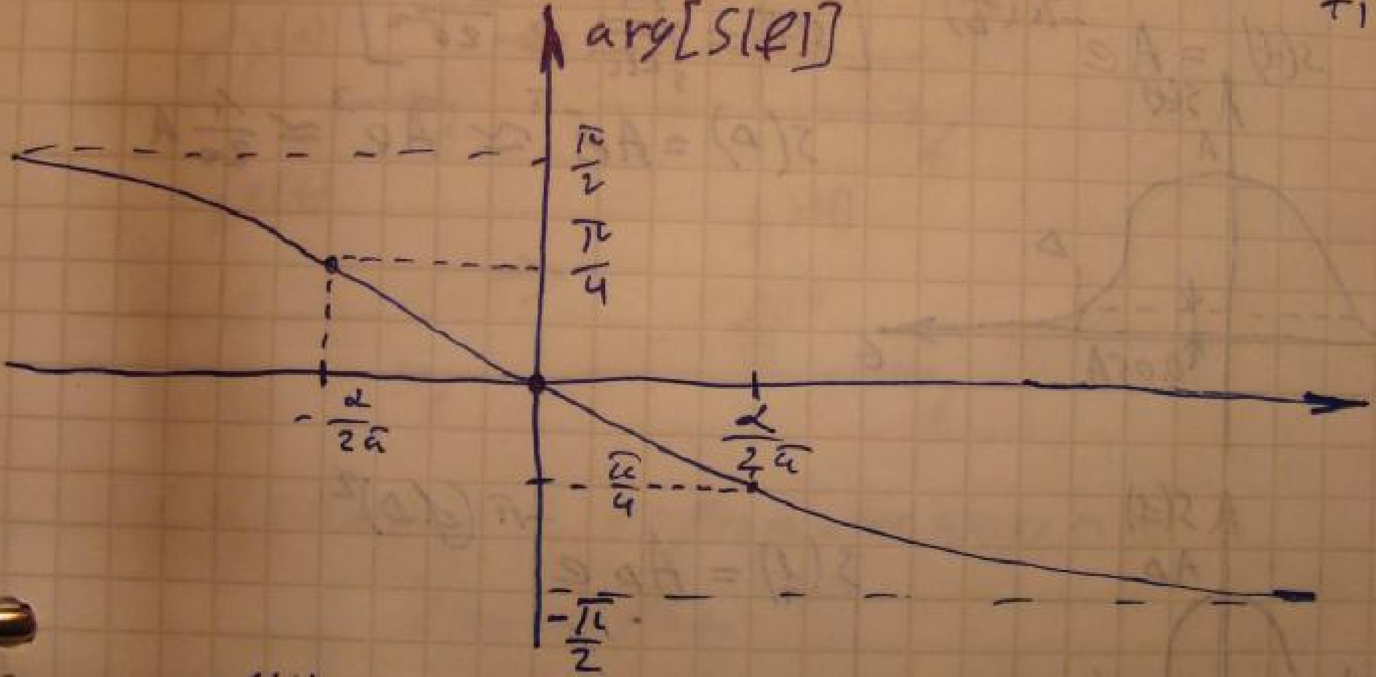
$$S(f) = \frac{A}{\alpha + j2\pi f}$$

$$S(t) = A\,e^{-\alpha t}$$

$$|S(f)| = \frac{A}{\sqrt{\alpha^2 + (2\pi f)^2}}$$

$$\arg[S(f)] = 0 - \operatorname{arctg} \frac{2\pi f}{\alpha}$$





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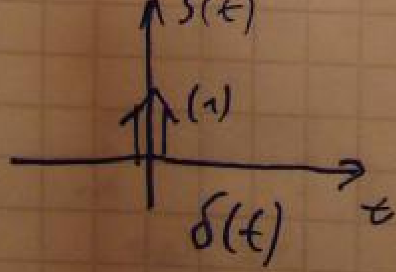

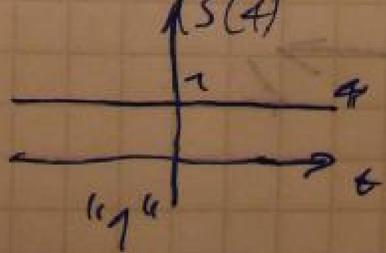

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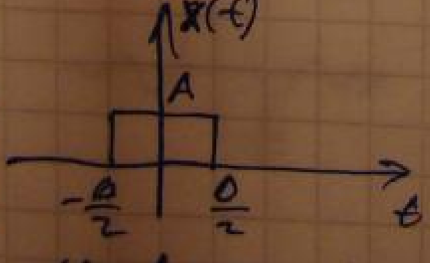
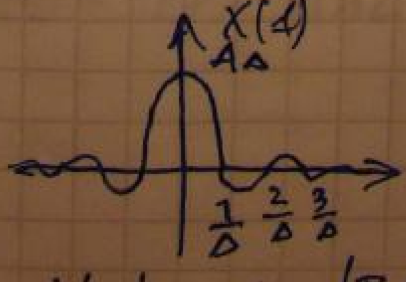
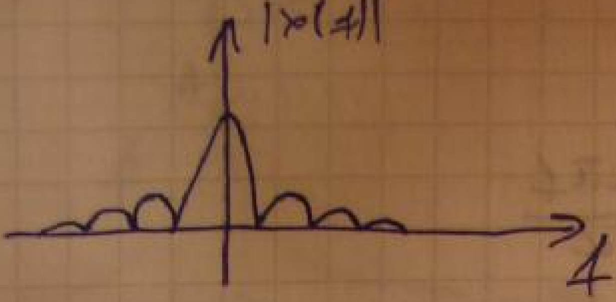
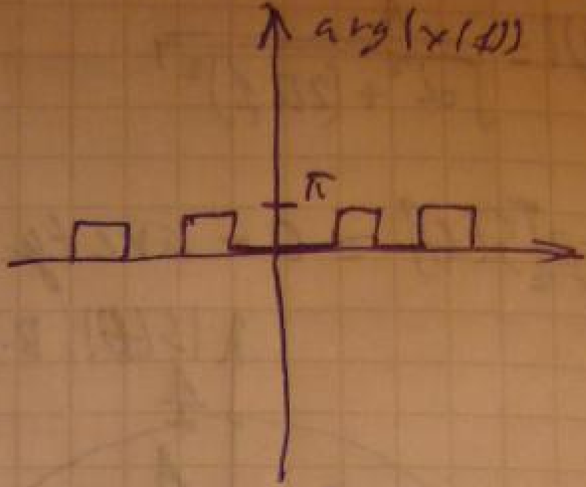




$x(t) = A\,\mathrm{rect}_\Delta(t)$ $\quad X(f) = \mathrm{sinc}(\pi f \Delta)\, A\Delta$

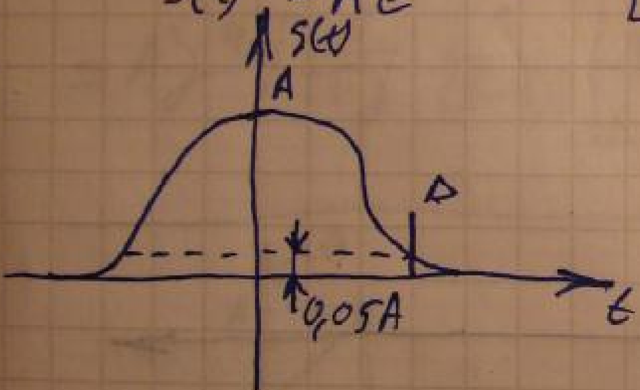




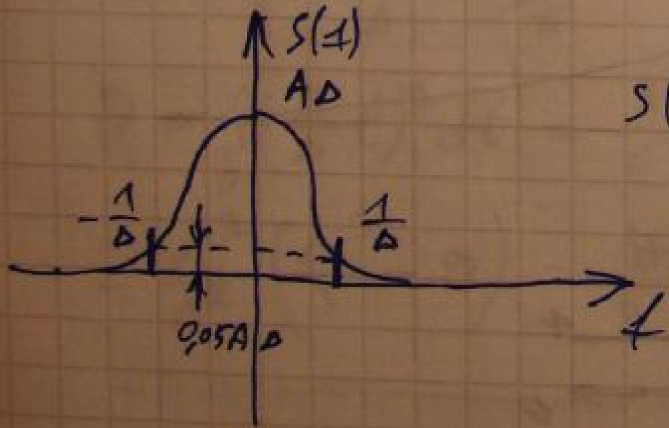
3) $y(t) = A e^{-\alpha t} u(t)$ | $Y(f) = \dfrac{A}{\alpha + j2\pi f}$

4) "Гаусс"

$$s(t) = A e^{-\pi \left(\frac{t}{\Delta}\right)^2} \qquad \left[s(t) = \frac{M}{\sqrt{2\pi}} e^{-\frac{t^2}{2\sigma^2}}\right]$$



$$s(\Delta) = A e^{-\pi} \simeq A e^{-3} \approx \frac{1}{20} A$$



$$S(f) = A\Delta e^{-\pi (f\Delta)^2}$$

Лекция 2 11.09

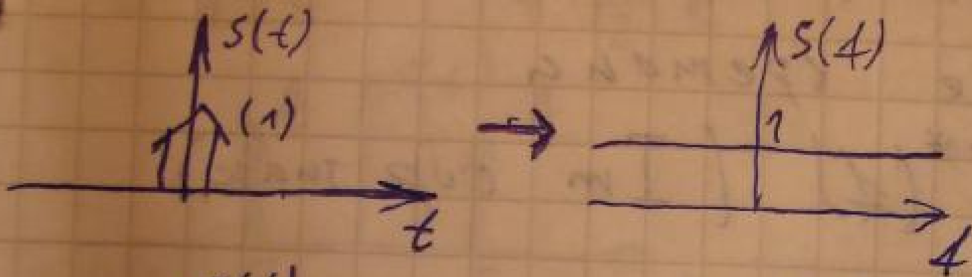
Свойства преобразования Фурье

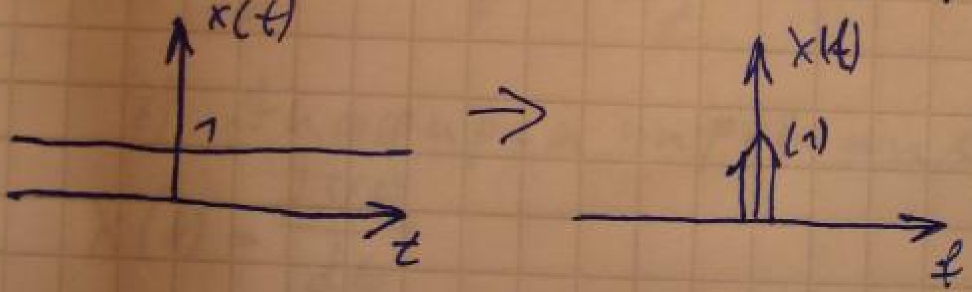
Свойство дуальности

$s(t) \to S(f)$

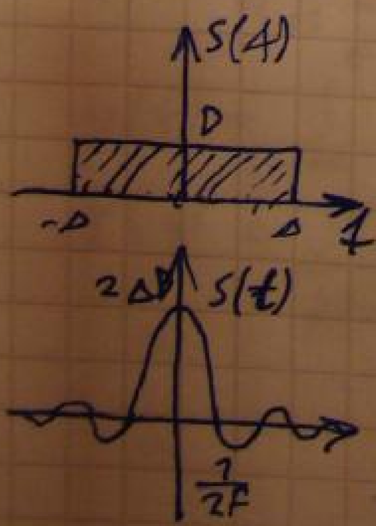
$x(t) \Leftrightarrow S(f)$

$X(f) = -s(t)$





Сигнал с ограниченным спектром



$s(t) = 2\Delta F \operatorname{sinc}(\pi t 2F)$

Основные свойства преобр. Фурье

$s(t) \Leftrightarrow S(f)$

1) $\alpha s(t) \Leftrightarrow \alpha S(f)$

2) $s_1(t) + s_2(t) \Leftrightarrow S_1(f) + S_2(f)$

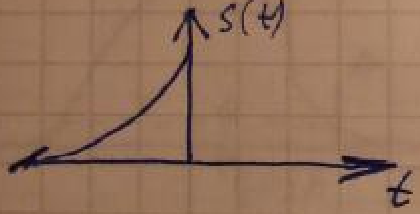
1) + 2) – свойство линейности

$a(t) = \alpha s(t) + \beta x(t)$

$A(f) = \alpha S(f) + \beta X(f)$

3) Обращение времени

$s(-t) \Leftrightarrow S^*(f)$ (Im обр знак)

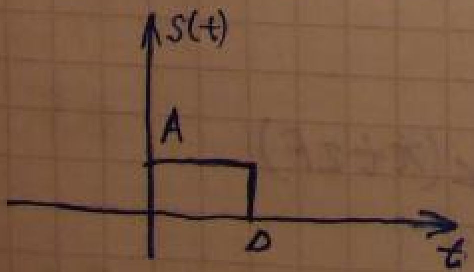


$s(t) = A e^{\alpha t} u(-t)$

$S(f) = \left(\frac{A}{\alpha + j2\pi f}\right)^* = \frac{A}{\alpha - j2\pi f}$

4) Задержка во времени

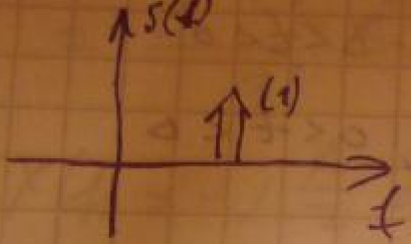
$s(t-\tau) \Leftrightarrow S(f) e^{-j2\pi f \tau}$



$S(f) = A\Delta \operatorname{sinc}(\pi f \Delta) \cdot e^{-j2\pi f \frac{\Delta}{2}}$

5) Смещение по частоте

$$s(t)\, e^{j2\pi Ft} \Leftrightarrow S(f-F)$$



$$y(t) = A\cos(2\pi F t) = A\,\frac{e^{-j\pi F t 2} + e^{j\pi F t 2}}{2} =$$

$$= \frac{A}{2} e^{-j2\pi F t} + \frac{A}{2} e^{j2\pi F t}$$

6) Диффиренцирование

$$x(t) = \frac{ds(t)}{dt}$$

$$X(f) = S(f)\, j2\pi f$$

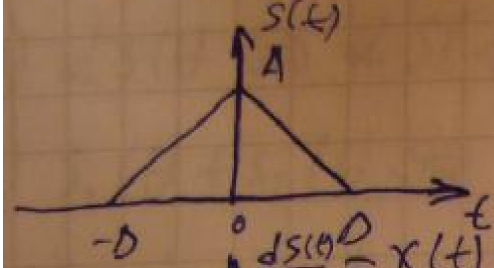
7) Интегрирование

$$y(t) = \int_{-\infty}^{t} s(\tau)\, d\tau$$

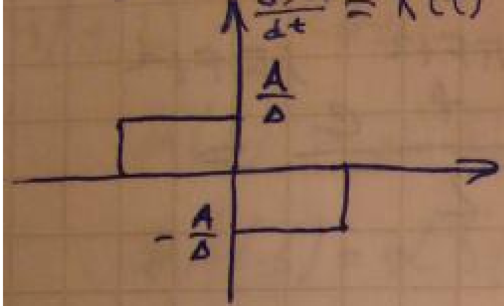
$$Y(f) = \frac{S(f)}{j2\pi f} + \frac{S(0)}{2}\,\delta(f)$$

Примера:

Спектр треугольника



$$s(t)=\begin{cases}\frac{A}{\Delta}(t+\Delta), & -\Delta<t<0\\ -\frac{A}{\Delta}t, & 0<t<\Delta\end{cases}$$



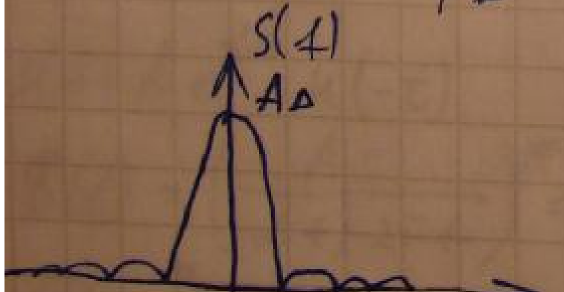
$$x(t)=y\left(t+\frac{\Delta}{2}\right)-y\left(t-\frac{\Delta}{2}\right)$$

$$X(f)=A\,\mathrm{sinc}(\cancel{2}\pi f\Delta)\,e^{-j2\pi f\frac{\Delta}{2}}=$$

$$=A\,\mathrm{sinc}(\cancel{2}\pi f\Delta)\left(e^{j\pi f\Delta}-e^{+j\pi f\Delta}\right)$$

$$S(f)=X(f)/j2\pi f=\frac{A\,\mathrm{sinc}(\pi f\Delta)\,2j\sin(\pi f\Delta)}{j2\pi f}\cdot\Delta$$

$$=\frac{A\Delta\,\mathrm{sinc}(\pi f\Delta)\sin(\pi f\Delta)}{\pi f\Delta}=A\,\mathrm{sinc}^2(\pi f\Delta)$$



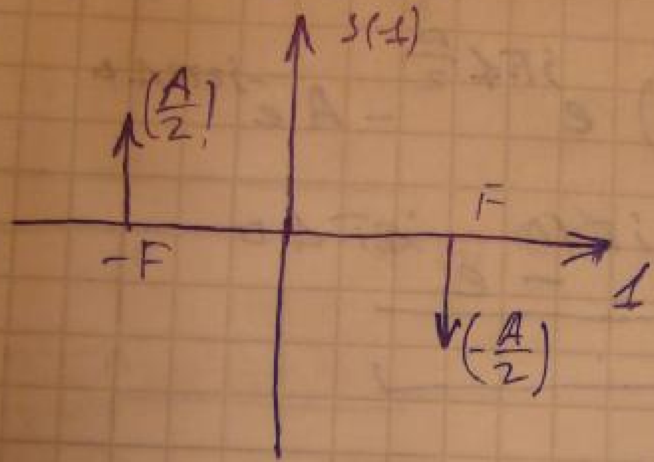
Спектр синуса

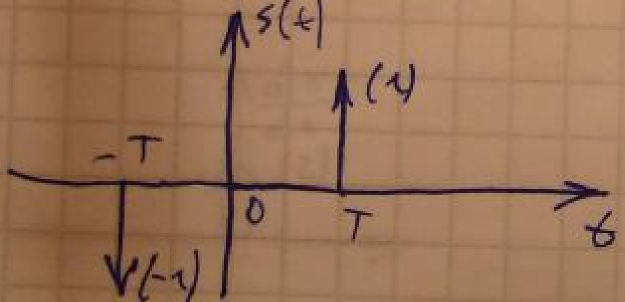
$$s(t) = A \sin(2\pi F t) =$$

$$= \frac{e^{j2\pi Ft} - e^{-j2\pi Ft}}{2j} = \frac{e^{j2\pi Ft}}{2j} - \frac{e^{-j2\pi Ft}}{2j}$$

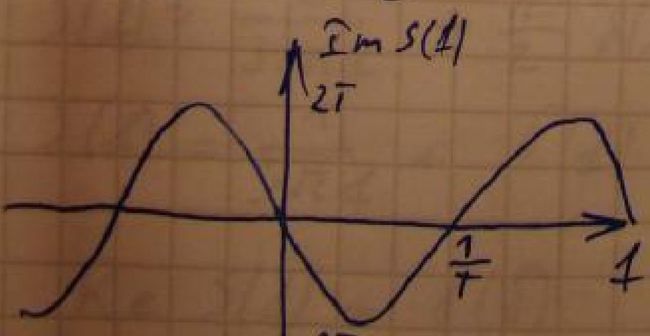
$$X(f) = -j\frac{A}{2}\delta(f-F) + j\frac{A}{2}(f+F)$$

$$\mathrm{Re}\, X(f) = 0$$



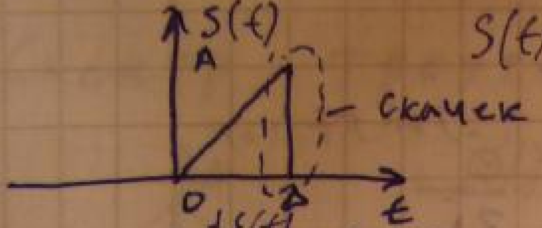


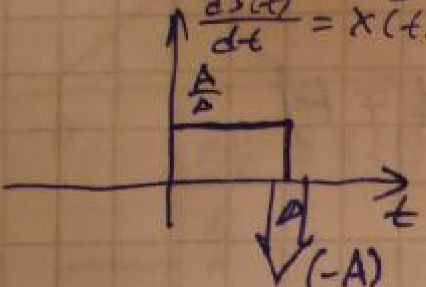
$$e^{j2\pi fT} + e^{-j2\pi fT}$$



Односторонний треугольник

$$s(t) = \frac{A}{\Delta} t, \quad 0 < t < \Delta$$



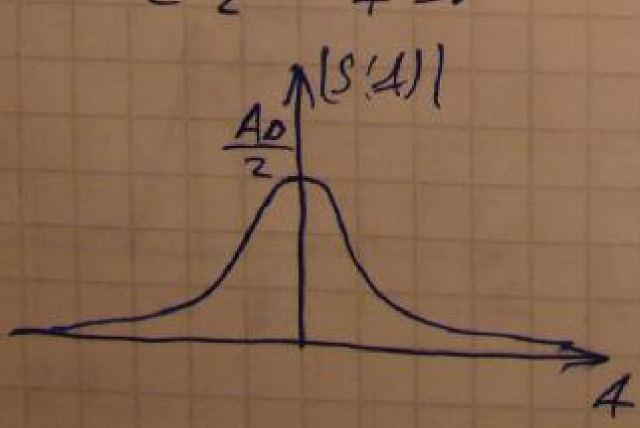


$$X(f) = \frac{A}{\Delta} \operatorname{sinc}(\pi f \Delta)\, e^{j\pi f \frac{\Delta}{2}} - A e^{-j2\pi f \Delta}$$

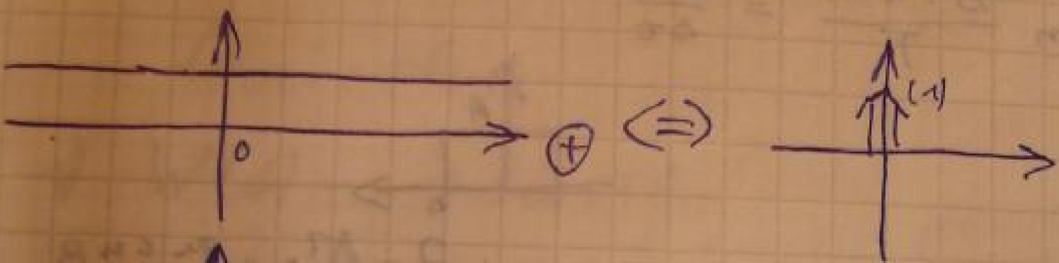
$$S(f) = \underbrace{\frac{A \operatorname{sinc}(\pi f \Delta)\, e^{-j\pi f \Delta} - e^{-j2\pi f \Delta}}{2j\pi f}}_{**}$$

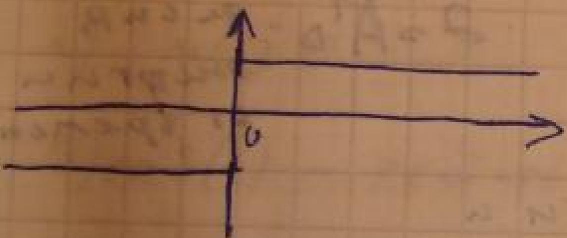
$$S(0) = \frac{A\Delta}{2}$$

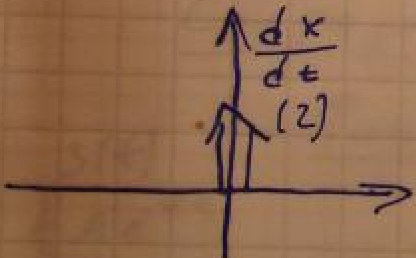
$$S(f) = \begin{cases} ** & f \neq 0 \\ \frac{A\Delta}{2} & f = 0 \end{cases}$$



$$s(t) = \begin{cases} 1, & t > 0 \\ -1, & 0 > t \\ 0, & t = 0 \end{cases} \iff S(f) = \frac{1}{j\pi f}$$





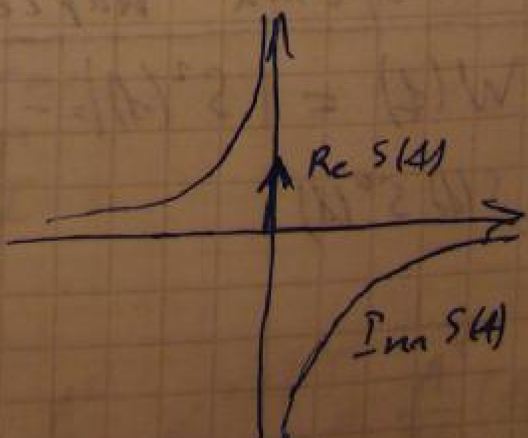


$$S(f) = \frac{2}{j2\pi f} + \frac{2}{2}\delta(f)$$

$$S(f) = \frac{1}{j\pi f} + \delta(f)$$

$$\underline{\mathrm{Re}}\, S(f) = \delta(f)$$

$$\underline{\mathrm{Im}}\, S(f) = -\frac{1}{j\pi f}$$



Энергетический спектр

$s(t)$

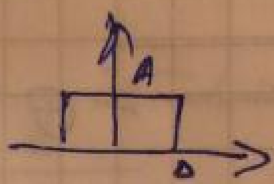
$$Э=\int_{-\infty}^{+\infty}|s^2(t)|dt$$

$p(t)=s^2(t)$ - мгновенная мощность

$p(t)=[В^2]$

$$p=\frac{Э}{t}$$

$$p(t_0)=\lim\frac{\Delta Э(t_0)}{T}=\frac{\Delta Э}{\Delta t}$$



$Э=A^2\Delta$ - распр. энергии во времени

Распределение энергии

в области частот

$s(t)\Leftrightarrow S(f)$

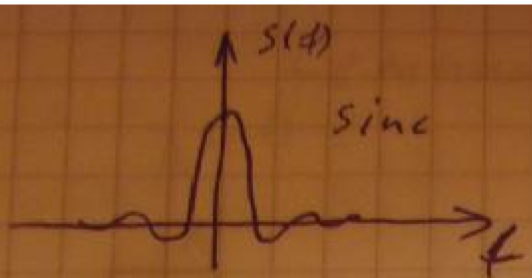
$$\boxed{Э=\int_{-\infty}^{+\infty}s^2(t)dt=\int_{-\infty}^{+\infty}|S(f)|^2df}$$

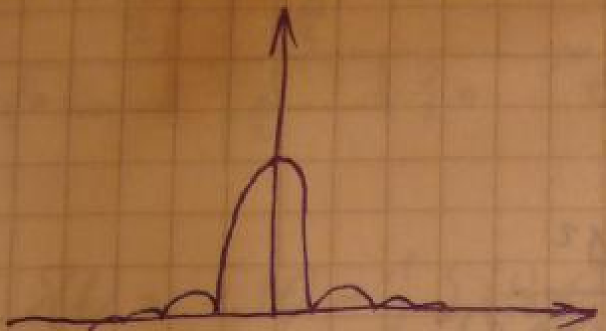
Теорема Парсеваля

$W(f)=|S^2(f)|=$ - Энергетический спектр

$=S(f)S^*(f)$

(спектральная плотность энергии)





$$Э = \int_{-\infty}^{+\infty} W\,dt = \frac{A^2 D^2 + \frac{2}{D}}{2} = A_0^2$$

Свойства

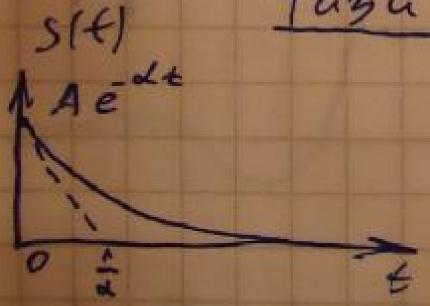
1) $W(t)$ – действ.

2) $W \geq 0$

3) если $s(t)$ – действ., $W(t) = ev(t)$

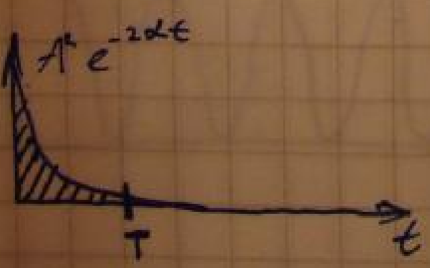
<u>Сосредоточенность энергии.</u>

<u>Физическая протяженность</u>

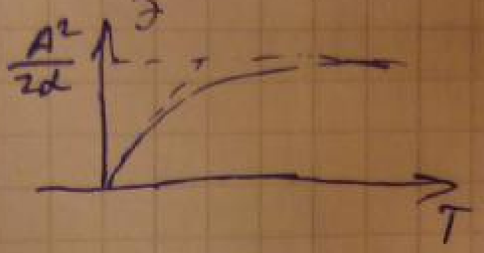


Критерий определения длительности по энергии

$$Э = \frac{A^2}{2\alpha}$$



$$Э_м(T) = \int_0^T A^2 e^{-2\alpha t}\,dt = \frac{A^2}{-2\alpha} e^{-2\alpha t}\Big|_0^T = \frac{A^2}{2\alpha}\left(1 - e^{-2\alpha T}\right)$$

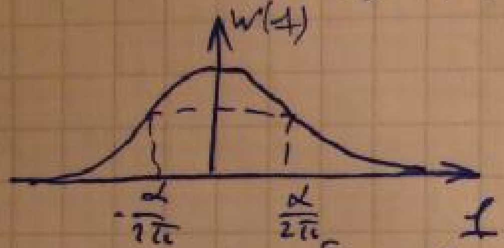


$$0{,}95\,Э = \frac{A^2}{2\alpha}\left(1 - e^{-2\alpha T}\right)$$

$$T = \frac{3}{2\alpha}$$

$$T_{95\%} = 1{,}5\,\frac{1}{\alpha}$$

$$W(f) = \left|\frac{A}{\alpha + j\pi f}\right|^2 = \frac{A^2}{\alpha^2 + (2\pi f)^2}$$



$$Э_F(F) = \int_{-F}^{F} \frac{A^2}{\alpha^2 + (2\pi f)^2}\,df = \qquad \int \frac{1}{1+x^2} = \operatorname{arctg}$$

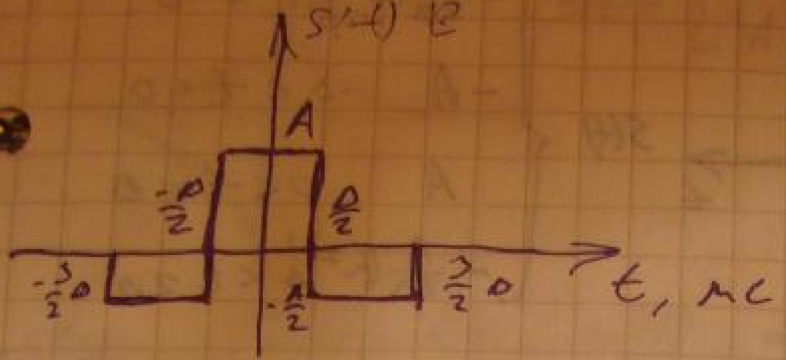
$$= \frac{A^2}{2\pi\alpha}\int_{-F}^{F} \frac{1}{1 + \left(\frac{2\pi}{\alpha} f\right)^2} = \frac{A^2}{2\pi\alpha}\operatorname{arctg}\left(\frac{2\pi f}{\alpha}\right)\Big|_{-F}^{F}$$

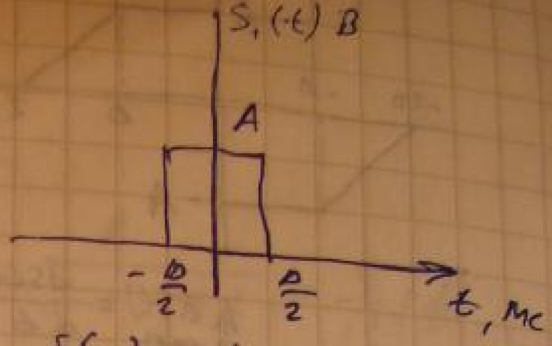
$$= \frac{A^2}{2\pi\alpha}\operatorname{arctg}\frac{2\pi F}{\alpha}$$

$$F = \frac{\alpha}{2\pi}\,\operatorname{tg}\frac{0{,}95\,\pi}{2} \approx 2\alpha$$

$T_{сигн}\,T_{спектр} = 3$ – база сигнала

Семинар 3. 15.09





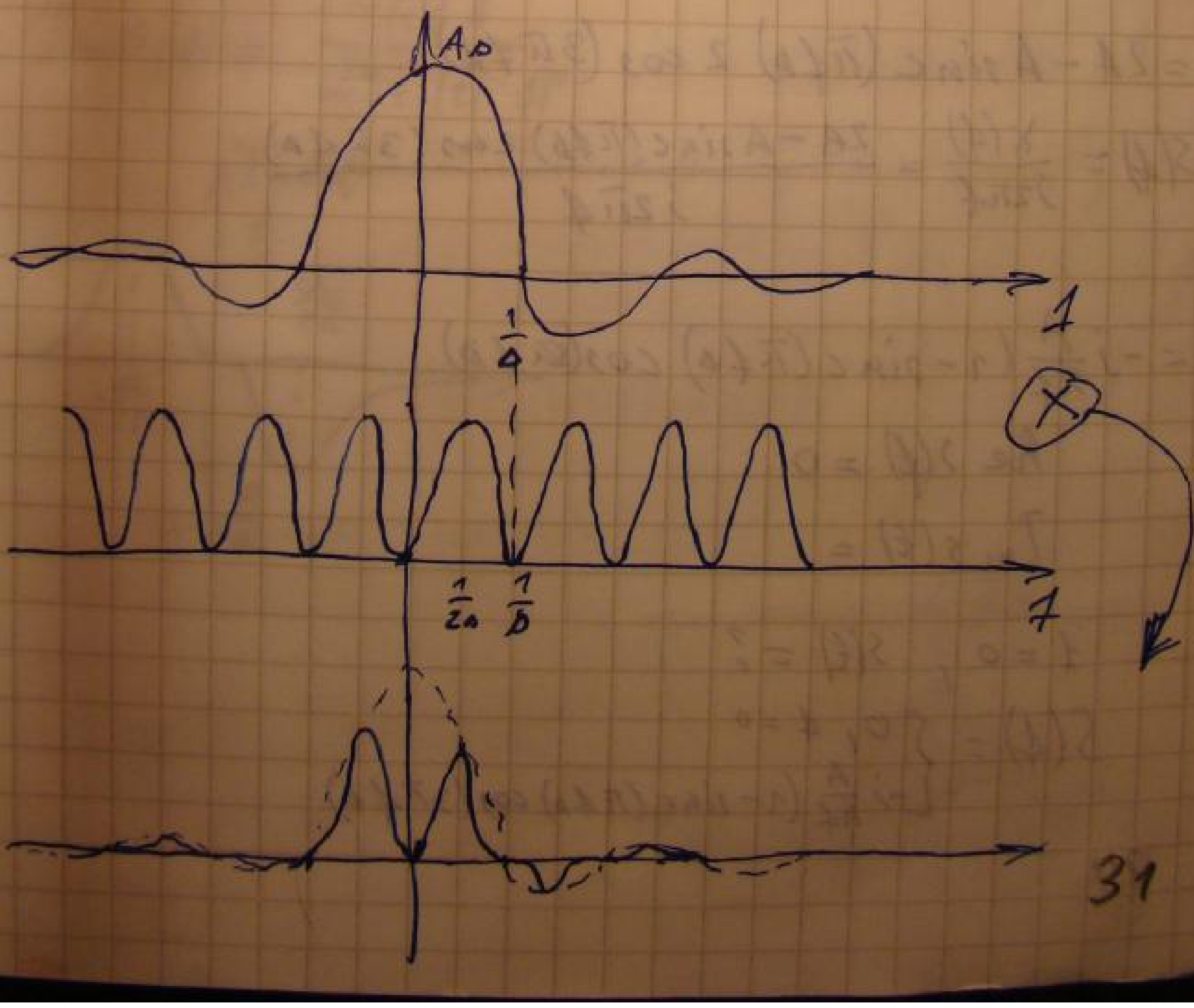
$$S(t) = S_1(t) - \frac{1}{2}S_1(t+\Delta) - \frac{1}{2}S_1(t+\Delta) \qquad S_1(f) = A\Delta\,\mathrm{sinc}(\pi f\Delta)$$

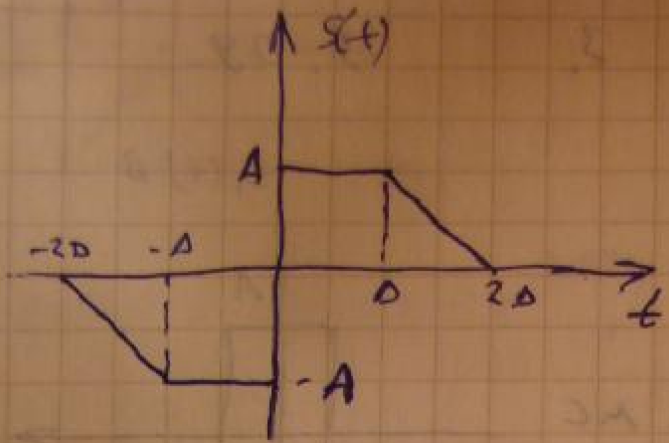
$$S(f) = A\Delta\,\mathrm{sinc}(\pi f\Delta) - \frac{1}{2}A\Delta\,\mathrm{sinc}(\pi f\Delta)e^{j2\pi\Delta f} -$$

$$- \frac{1}{2}A\Delta\,\mathrm{sinc}(\pi f\Delta)e^{-j2\pi\Delta f} =$$

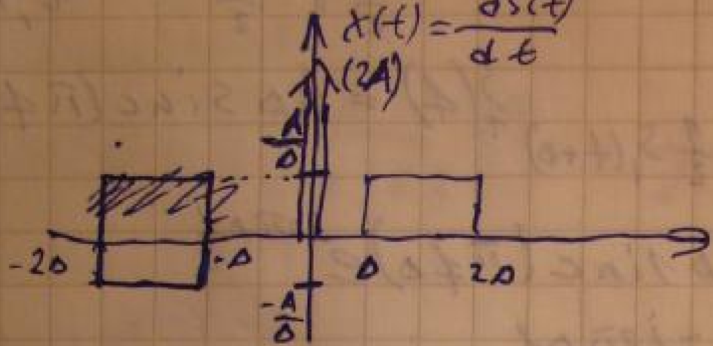
$$= A\Delta\,\mathrm{sinc}(\pi f\Delta)\cdot\left(1 - \frac{1}{2}e^{j2\pi\Delta f} - \frac{1}{2}e^{-j2\pi\Delta f}\right) =$$

$$= A\Delta\,\mathrm{sinc}(\pi f\Delta)\left(1 - \cancel{\tfrac{1}{2}}\cos(2\pi f\Delta)\right)$$





$$s(t)\begin{cases} -\frac{A}{2\Delta}(t+2\Delta) & -2\Delta<t<-\Delta \\ -A & -\Delta<t<0 \\ A & 0<t<\Delta \\ -\frac{A}{2\Delta}(t-2\Delta) & \Delta<t<2\Delta \end{cases}$$



$$X(f) = 2A - \frac{A}{\Delta}\Delta\, \text{sinc}(\pi f\Delta)\, e^{j2\pi f\Delta\frac{3}{2}} - \frac{A}{\Delta}\Delta\, \text{sinc}(\pi f\Delta)\, e^{-j\ldots}$$

$$= 2A - A\,\text{sinc}(\pi f\Delta)\left(e^{j\cancel{2}\pi f\Delta\frac{3}{\cancel{2}}} + e^{-j\cancel{2}\pi f\Delta\frac{3}{\cancel{2}}}\right) =$$

$$= 2A - A\,\text{sinc}(\pi f\Delta)\, 2\cos(3\pi f\Delta)$$

$$S(f) = \frac{X(f)}{j2\pi f} = \frac{2A - A\,\text{sinc}(\pi f\Delta)\, 2\cos(3\pi f\Delta)}{j2\pi f}$$

$$= -j\frac{A}{\pi f}\left(1 - \text{sinc}(\pi f\Delta)\cos(3\pi f\Delta)\right)$$

$$\text{Re}\, S(f) = 0$$

$$\text{Im}\, s(t) =$$

$$f = 0, \quad S(f) = ?$$

$$S(f) = \begin{cases} 0, & f=0 \\ -j\frac{A}{\pi f}\left(1 - \text{sinc}(\pi f\Delta)\cos(3\pi f\Delta)\right) \end{cases}$$

$$s(t) = \begin{cases} -Ae^{-\alpha t}, & t<0 \\ Ae^{-\alpha t}, & t>0 \end{cases}$$

$-S_1^*(f)$

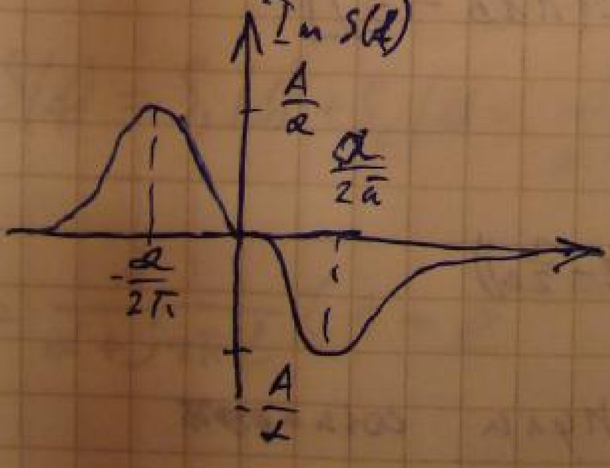
$$s_1(t) = Ae^{-\alpha t}u(t) \Rightarrow \frac{A}{\alpha + 2j\pi f} = S_1(f)$$

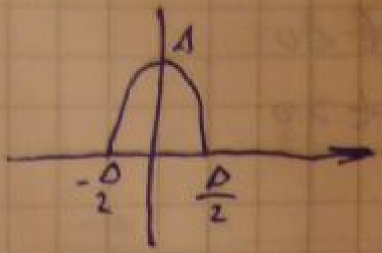
$$S_2(f) = -\frac{A}{\alpha - j2\pi f}$$

$$S(f) = \frac{A}{\alpha + 2j\pi f} - \frac{A}{\alpha - j2\pi f} = \frac{A(\alpha - j2\pi f) - A(\alpha + j2\pi f)}{\alpha^2 - (j2\pi f)^2} =$$

$$= \frac{-4j\pi f A}{\alpha^2 + (2\pi f)^2}$$

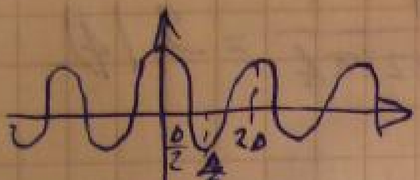
$$\operatorname{Im} S(f) = \frac{-4\pi f A}{\alpha^2 + (2\pi f)^2}$$



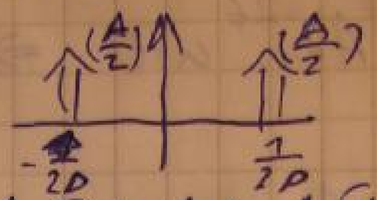


$$s(t)=\begin{cases}A\cos\left(2\pi \frac{t}{2\Delta}\right), & -\frac{\Delta}{2}<t<\frac{\Delta}{2}\\ 0, & \text{ост. } t\end{cases}$$

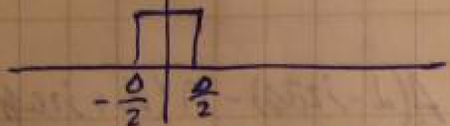
$$s(t)\cdot x(t) \Leftrightarrow S(f) * X(f)$$



$\Leftrightarrow$



$$\frac{A}{2}\delta\left(f+\frac{1}{2\Delta}\right)+\frac{A}{2}\delta\left(f+\frac{1}{2\Delta}\right)$$



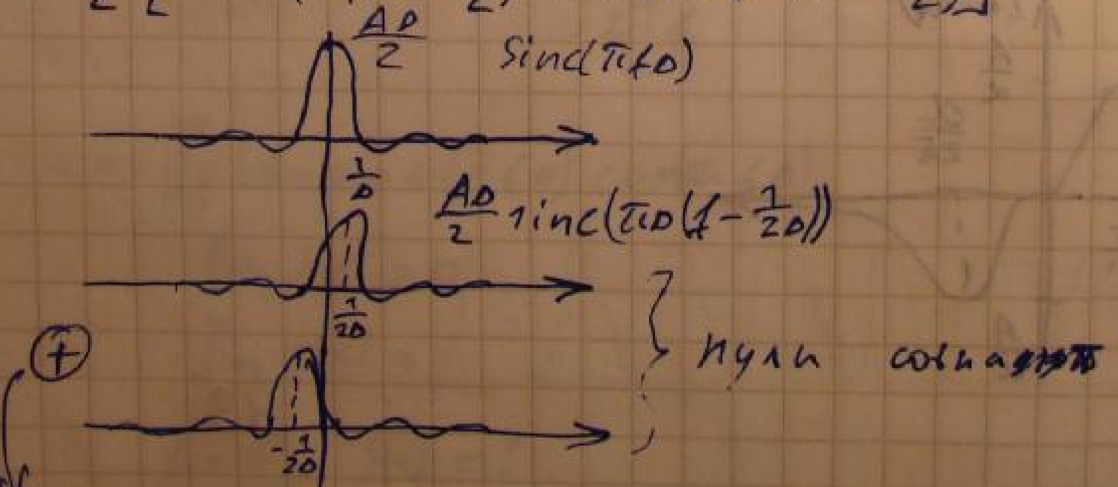
$\Leftrightarrow \Delta\cdot \mathrm{sinc}(\pi f\Delta)$

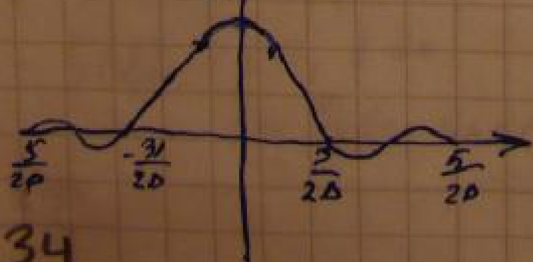
$$\Delta\,\mathrm{sinc}(\pi f\Delta) * \frac{A}{2}\left(\delta\left(f+\frac{1}{2\Delta}\right)+\delta\left(f-\frac{1}{2\Delta}\right)\right)=$$

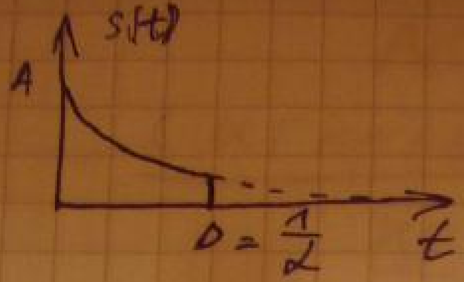
$$S(f)*\delta(f-F) = S(f-F)$$

$$=\frac{A}{2}\Delta\,\mathrm{sinc}(\pi f\Delta)*\delta\left(f+\frac{1}{2\Delta}\right)+\frac{A}{2}\Delta\,\mathrm{sinc}(\pi f\Delta)*\delta\left(f-\frac{1}{2\Delta}\right)$$

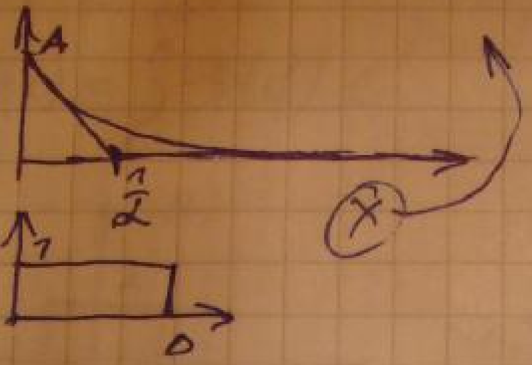
$$=\frac{A\Delta}{2}\left[\mathrm{sinc}\left(\pi f\Delta+\frac{\pi}{2}\right)+\mathrm{sinc}\left(\pi f\Delta-\frac{\pi}{2}\right)\right]$$







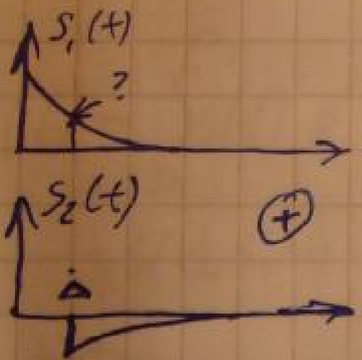
$$s(t) = A e^{-\alpha t} \quad 0<t<\Delta$$



$$\frac{A}{\alpha + j2\pi f}$$ ← свотн. интеграл.

$$*$$

$$\Delta\, \mathrm{sinc}(\pi f \Delta)\, e^{-j\pi f\Delta}$$



$$s_1(t) = A e^{-\alpha t}\, u(t)$$

$$s_2(t) = -A e^{-\alpha\Delta} e^{-\alpha(t-\Delta)}\, u(t-\Delta)$$

$$S_2(f) = \frac{-A e^{-\alpha\Delta}}{\alpha + j2\pi\Delta} e^{-j2\pi f\Delta}$$

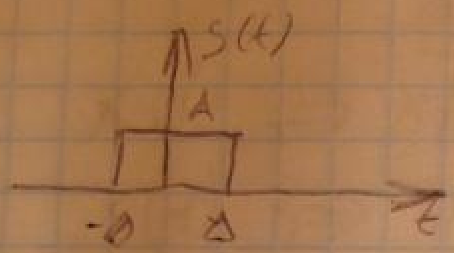
$$S(f) = S_1(f) + S_2(f) = \frac{A}{\alpha + j2\pi f}\left(1 - e^{-\alpha\Delta} e^{-j2\pi f\Delta}\right) =$$

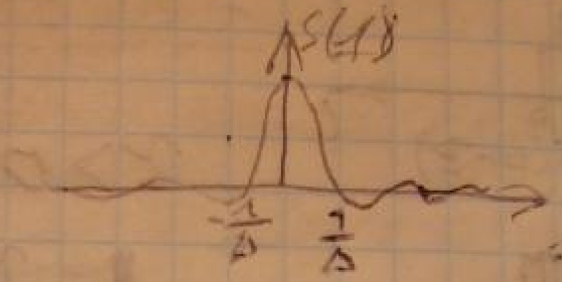
$$= \frac{A}{\alpha + j2\pi f}\left(1 - e^{-(\alpha + j2\pi f)\Delta}\right)$$

Лекция 3   18.09

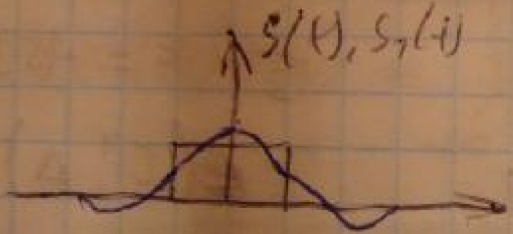
Сигналы восстановление по части спектра

$$S(t) = \int_{-\infty}^{+\infty} s(f)\, e^{j2\pi f t}\, df$$





$$S_1(t) = \int_{-D}^{D} s(f)\, e^{j2\pi f t}\, df$$

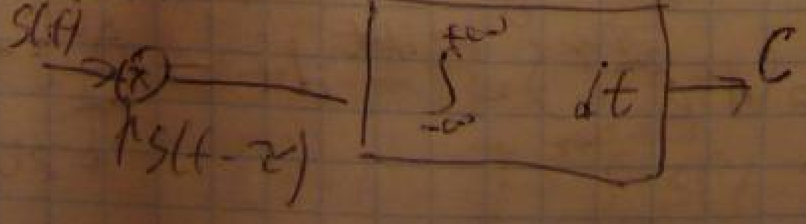


$$S(t) - S_1(t) = e(t)$$

$$\frac{Э(e(t))}{Э(s(t))} = \frac{\int_{-\infty}^{+\infty} (s(t) - s_1(t))^2\, dt}{\int_{-\infty}^{+\infty} s(t)^2\, dt}$$ – относительная средне квадр. ошибка

Автокорреляционная ф-я

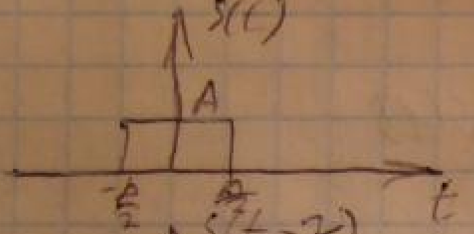
АКФ

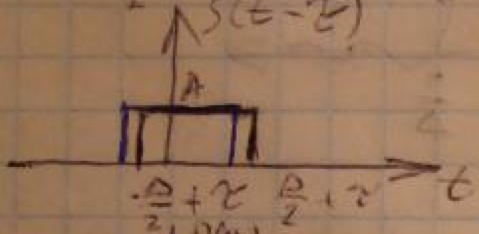


Ф-ия АКФ – ev(t) – чётная

$$R(\tau)=\int_{-\infty}^{+\infty} s(t)\,s(t+\tau)\,dt$$

Max в 0 = Э(s(t))

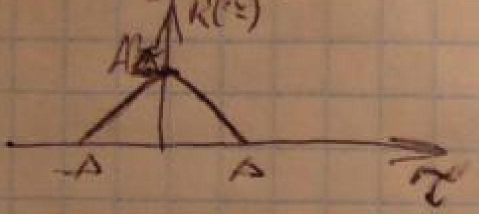




$\tau \geq 0$

$\frac{\Delta}{2} - \tau = 0$

$\frac{\Delta}{2} - \tau > 0$



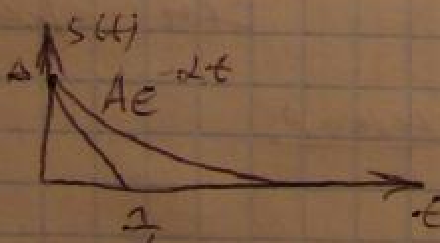
$$\int_{-\frac{\Delta}{2}+\tau}^{\frac{\Delta}{2}} s(t)\,s(t-\tau)\,dt = \int_{-\frac{\Delta}{2}+\tau}^{\frac{\Delta}{2}} A^2 dt = A^2(\Delta-\tau)$$

$$A^2(\Delta-|\tau|)=\begin{cases} A^2(\Delta-\tau) & \tau>0, \\ A^2(\Delta+\tau) & \tau<0 \end{cases}$$

$\tau \in (0;\Delta)$

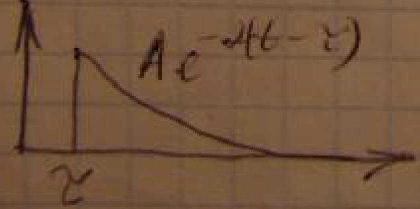
0 для $\tau \notin (0;$

$$R(\tau)=\begin{cases} A^2(\Delta-\tau), & \tau>0 \\ A^2(\Delta+\tau), & \tau<0 \\ 0, & \text{ост. } \tau \end{cases}$$



$$R_s(\tau)=\int_{\tau}^{0} A e^{-\alpha t} A e^{-\alpha(t-\tau)} u(t)\,u(t-\tau)\,dt$$

$$= A^2\int_{\tau}^{0} e^{-2\alpha t(t-\tau)} u(t-\tau)\,u(t)\,dt$$



$$= A^2 e^{2\alpha\tau}\int_{\tau}^{0} e^{-2\alpha t}\,dt = \frac{A^2}{2\alpha} e^{-\alpha t}$$

$t \geq 0$

$$\begin{cases} \frac{A^2}{2\alpha} e^{-\alpha\tau} & \tau>0 \\ \frac{A^2}{2\alpha} e^{\alpha\tau} & \tau<0 \end{cases}$$

Свойства произведения и свёртки

$$x(t) \Leftrightarrow X(f)$$

$$s(t) \Leftrightarrow S(f)$$

$$\boxed{\begin{aligned} Y(f) &= X(f) * S(f) \\ y(t) &= x(t)\, s(t) \end{aligned}}$$

$$X(f) * S(f) = \int_{-\infty}^{+\infty} X(\nu)\, S(f-\nu)\, d\nu$$

$$\boxed{\begin{aligned} y(f) &= X(f) \cdot S(f) \\ y(t) &= x(t) * s(t) \end{aligned}}$$

Свойства δ фу-ии

Свойство площади: $\int_{-\infty}^{\infty} \delta(t) = 1$

Свойство произв. $x(t)\, \delta_T(t) = x(T)\, \delta_T(t)$

И. свёртки.

$$s(t) * \delta(t-T) = \int_{-\infty}^{\infty} s(\tau)\, \delta(t-\tau-T)\, d\tau =$$

$$= \int_{-\infty}^{+\infty} s(\tau)\, \delta[(t-T)-\tau] = \int_{-\infty}^{+\infty} s(t-T)\, \delta[(t-T)-\tau]\, d\tau =$$

$$= s(t-T) \cdot \underbrace{\int_{-\infty}^{+\infty} \delta[(t-T)-\tau]\, d\tau}_{1} = s(t-T)$$

$$s(t) * \delta(t) \neq s(t-T)$$

Осн. свойства свёртки.

1) коммутативность

~~$s * x * y = s*(x*y) =$~~

$$s * y = y * s$$

2) ассоциативность

$$s*x*y = s*(x*y) = (s*y)*x$$

3) дистрибутивность

$$s*(x+y) = s*x + s*y$$

4) св. линейности

$$(\alpha A)*(\beta B) = (\alpha\beta)(A*B)$$

5) связь свёртки и акф

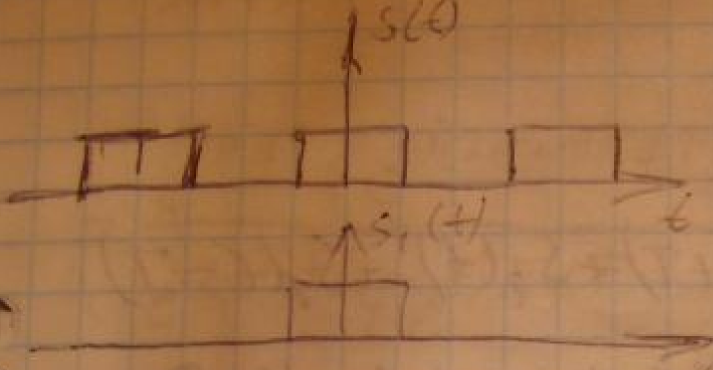
$$\text{акф}\{s(t)\} = \int_{-\infty}^{+\infty} s(t-\tau)\, dt$$

$$s(\tau) * s(\tau) = \int_{-\infty}^{+\infty} s(t)\, s(\tau - t)\, dt$$

$$\text{акф}\{s(t)\} = R(\tau) = s(\tau) * s(-\tau)$$

Периодические сигналы и их спектры

1. Пачка импульсов.

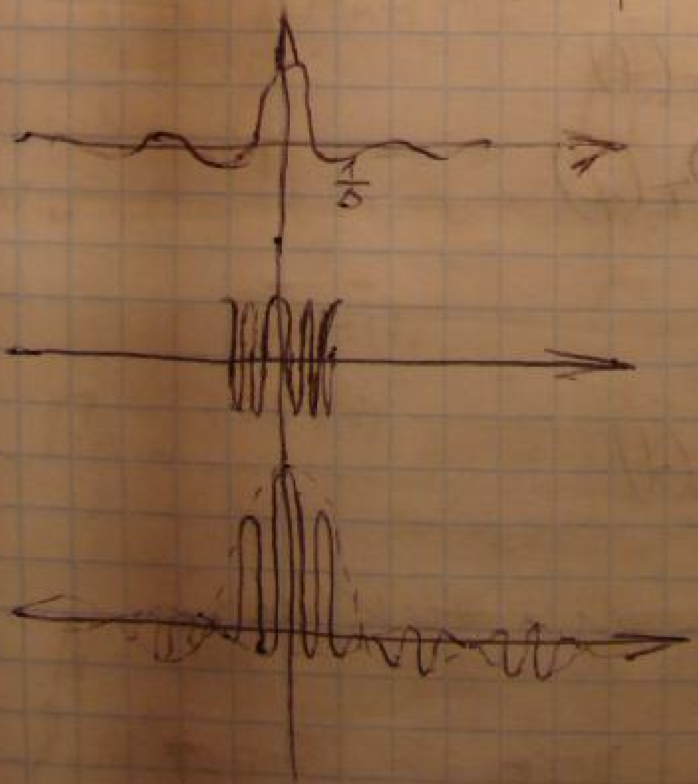


$$s(t) = s_1(t-T) + s_1(t) + s_1(t+\ldots$$
$$= s_1(t) + s_1(t)\,\delta(t+T) +$$
$$+ s_1(t)\,\delta(t-T) =$$
$$= s_1(t) * \underbrace{(\delta(t) + \delta(t+T) + \delta\ldots}_{d_3(t) = \text{Ш}_3(t)}$$

$$\Rightarrow S(f) = S_1(f) \cdot D_3(f)$$

$$A\Delta\, \mathrm{sinc}(\pi f \Delta)\left(1 + e^{-j2\pi f T} + e^{j2\pi f T}\right) =$$
$$= S(f)$$

$$S(f) = A\Delta\, \mathrm{sinc}(\pi f \Delta)\left(1 + 2\cos(2\pi f t)\right)$$



Асинх лёд 301 ГАК

Формирование периодич. сигнала

$s(t)$, $s(t) = s(t+kT)$ $\forall t$

$s_T(t)$

↑ указ. период.

$$s(t) = s_1(t+2T) + s_1(t+T) + s_1(t) + s_1(t-T)$$

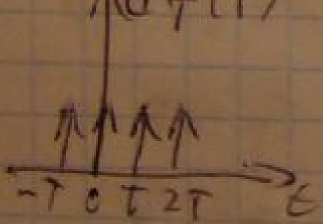
$s_1(t)$ — один. сигнал из кот. мож. форм. периодич

$$s_T(t) = \sum_{k=-\infty}^{+\infty} \delta(t-kT)$$

$$s(T) = s_1(t) * \sum_{k=-\infty}^{+\infty} \delta(t-kT)$$ б посл.

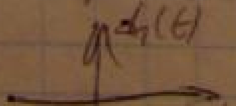
$$s_T(t) = s_1(t) * d_T(t)$$

$$S_T(f) = S_1(f) \cdot D_T(f)$$

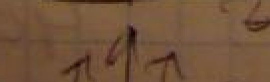


$d_T(t) = Ш_T(t)$

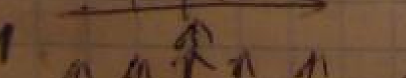
Метод мат. индукции



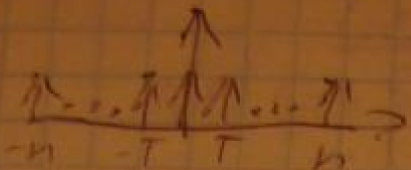
$\Leftrightarrow 1$



$\Leftrightarrow 1 + e^{j2\pi fT} + e^{-j2\pi fT}$



$\Leftrightarrow 1 + e^{j2\pi fT} + e^{-j2\pi fT} + e^{j2\pi f2T} + e^{-j2\pi f2T}$



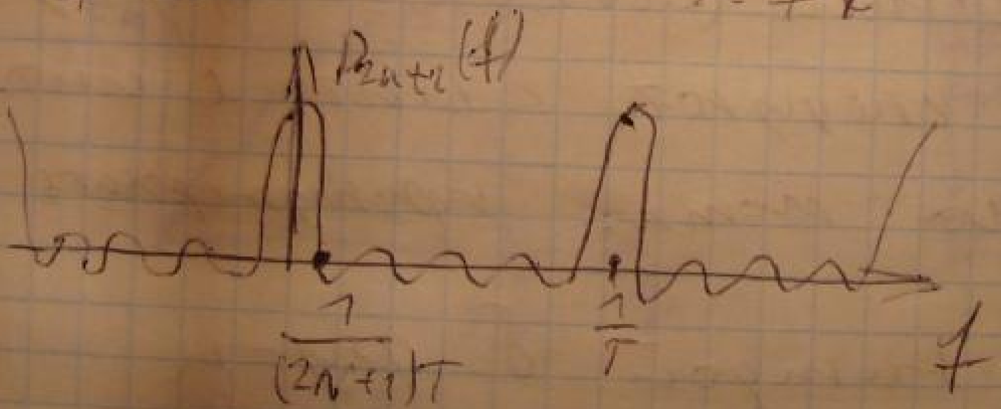
$\Leftrightarrow 1 \dots e^{j2\pi f n T} + e^{-j2\pi f n T}$

$$D_{2N+1}(f) = \sum_{n=-N}^{+N} e^{j2\pi f n T} = \frac{e^{j2\pi f n T} \pm e^{\pm j2\pi f (N+1) T}}{1 - e^{j2\pi f T}} =$$

$$= \frac{e^{j\pi f T}\left(e^{+j(2N+1)\pi f T} - e^{-j(2N+1)\pi f T}\right)}{e^{-j2\pi f T}\left(e^{+j\pi f T} - e^{-j\pi f T}\right)} =$$

$$= \frac{\sin[(2N+1)\pi f T]}{\sin(\pi f T)}.$$

$$\lim_{f \to 0} D_{2N+1}(f) = \frac{(2N+1)\pi f T}{\pi f T} = 2N+1$$



$$\Pi_n = \frac{(2N+1)\left(\frac{2}{2N+1}\right)T}{2} = \frac{1}{T}$$

$N \to \infty$, $\to \delta(f)$

$$D_T(f) = \sum_{n=-\infty}^{+\infty} \frac{1}{T}\delta\left(f - \frac{n}{T}\right)$$

$$\sum_{n=-\infty}^{+\infty} \delta(t - nT) \Leftrightarrow \sum_{n=-\infty}^{+\infty} \frac{1}{T}\delta\left(f - \frac{k}{T}\right)$$

$S_1(f)\, P_T(f) = S_T(f)$

$$S_T(f) = S_1(f) \sum_{m=-\infty}^{+\infty} \frac{1}{T}\delta\left(f - \frac{m}{T}\right) =$$

$$= \sum_{m=-\infty}^{+\infty} \frac{1}{T} S_1(f)\, \delta\left(f - \frac{m}{T}\right) =$$

$$= \sum_{m=-\infty}^{+\infty} \frac{S_1\left(\frac{m}{T}\right)}{T} \delta\left(f - \frac{m}{T}\right) = \sum_{m=-\infty}^{+\infty} S_T[m]\, \delta\left(f - \frac{m}{T}\right)$$

Дискр. ф-я $S_T[m]$

коэф. ряда Фурье $S_T[m] = S_m = \frac{S_1\left(\frac{m}{T}\right)}{T} = C_m$

1) Спектр периодич. сигнала дискретный (сост. из δ ф-й)

2) δ-ф-и следуют с инт. с инт. $\frac{\Delta}{T}$ их можно пронумеровать

3) Площ. каждой δ ф-и опр. коэф. ряда Фурье со своим номером

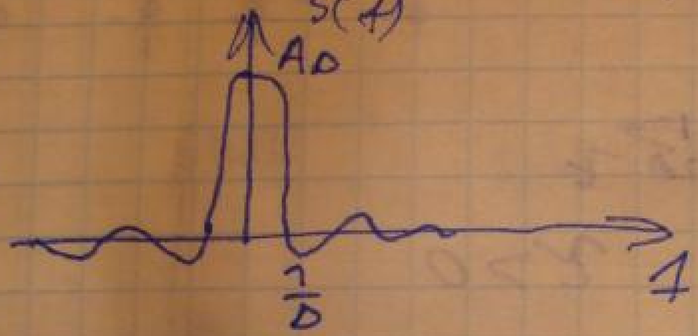
$$C_m = S_T[m] = \frac{1}{T} S_1\left(\frac{m}{T}\right) \quad (РФ)$$

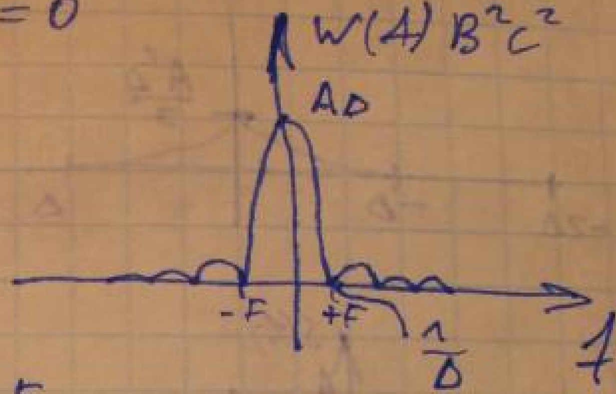
Семинар 4.

$$s(t) \Leftrightarrow S(f)$$

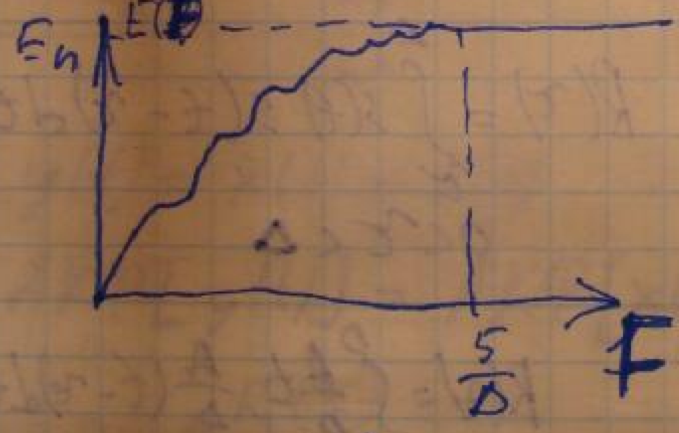
$$W(f) = |S(f)|^2$$

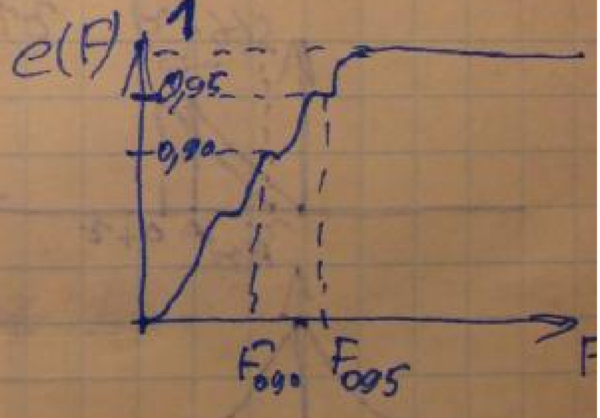
$$S(f) = \begin{cases} \dfrac{\sin(\pi f \Delta)}{\pi f \Delta}, & f \neq 0 \\ A\Delta, & f = 0 \end{cases}$$





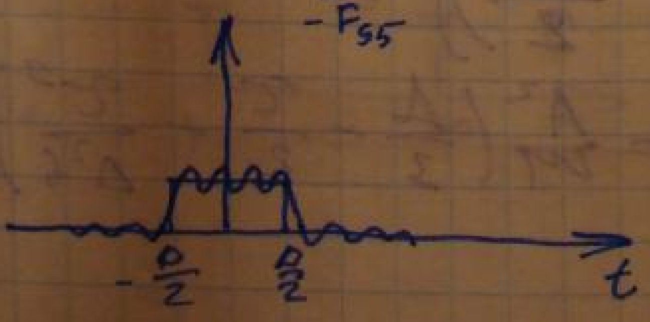
$$E(f) = \int_{-F}^{F} W(f)\,df = 2\int_{0}^{F} W(f)\,df$$

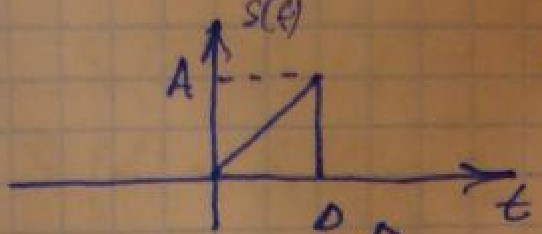




$$e(F) = \frac{E(F)}{Э}$$

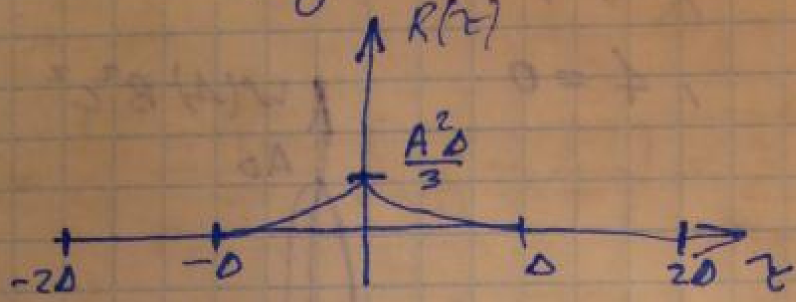
$$S_{95}(t) = \int_{-F_{95}}^{+F_{95}} S(f)\, e^{+j2\pi f t}\, df$$



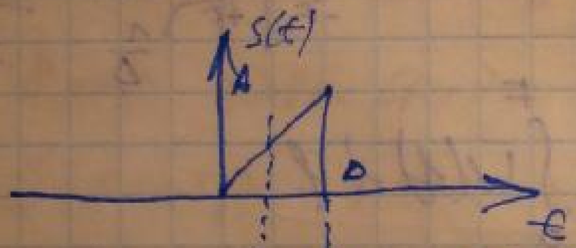


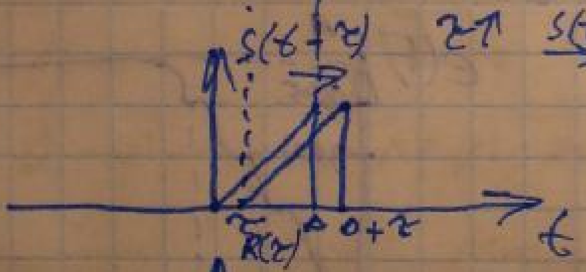
$$s(t) = \begin{cases} \frac{A}{\Delta} t, & 0 < t < \Delta \\ 0, & \text{ост. } t \end{cases}$$

$$R(\tau) = \int_{\cancel{-\infty}\ 0}^{\cancel{+\infty}\ \Delta} s(t)\, s(t-\tau)\, dt$$



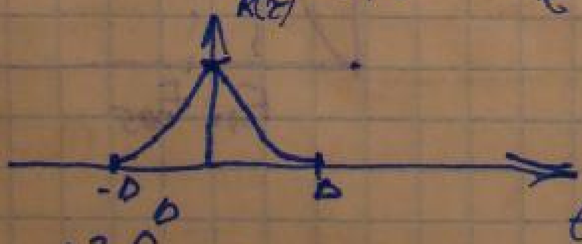
$\tau > 0$





$$R(\tau) = \int_{\tau}^{\Delta} s(t)\, s(t-\tau)\, dt$$

$0 < \tau < \Delta$



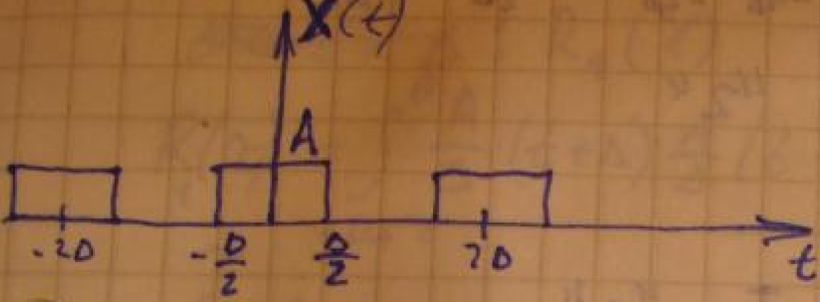
$$R(\tau) = \int_{\tau}^{\Delta} \frac{A}{\Delta} t\, \frac{A}{\Delta}(t-\tau)\, dt =$$

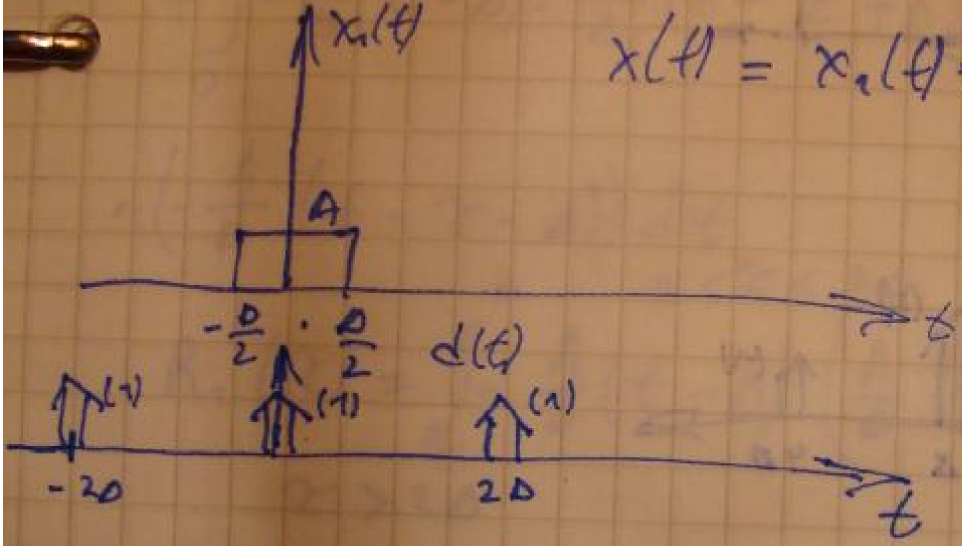
$$= \frac{A^2}{\Delta^2} \int_{\tau}^{\Delta} (t^2 - t\tau)\, dt = \frac{A^2}{\Delta^3} \left(\frac{t^3}{3} - \frac{t^2 \tau}{2} \right) \Bigg|_{\tau}^{\Delta} =$$

$$= \frac{A^2}{\Delta^2} \left(\frac{\Delta^3}{3} - \frac{\Delta^3 \tau}{3} - \left(\frac{\tau^3}{3} - \frac{\tau^3}{2} \right) \right) =$$

$$= \frac{A^2}{\Delta^2} \left(\frac{\Delta^3}{3} - \frac{\Delta^2 \tau}{2} + \frac{\tau^3}{6} \right) = \frac{A^2}{\Delta} \left(\frac{\Delta}{3} - \frac{\tau}{2} + \frac{\tau^3}{\Delta^2 6} \right)$$

$$R(\tau) = \begin{cases} A^2\left(\frac{\Delta}{3} - \frac{|\tau|}{2} + \frac{|\tau|^3}{6\Delta^2}\right), & 0 \leq |\tau| < \Delta \\ 0, & |\tau| \geq 0 \end{cases}$$

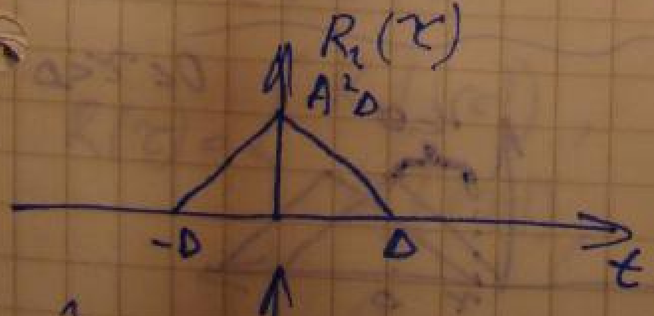


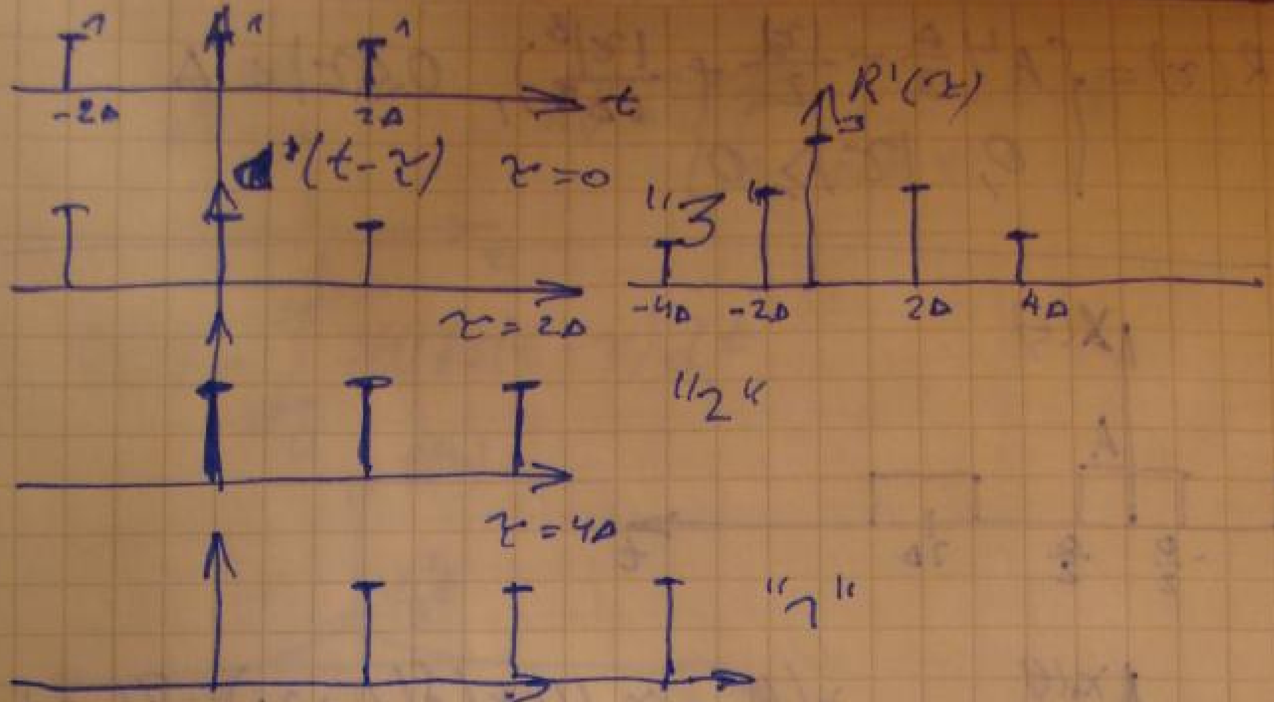


$$x(t) = x_1(t) * (\delta(t+2\Delta) + \delta(t-2\Delta) +$$

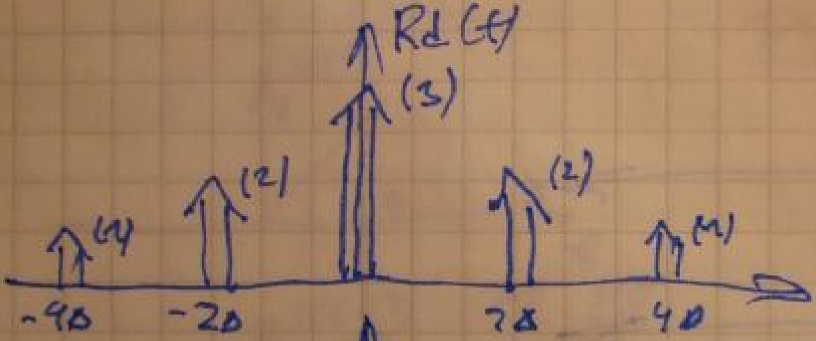
$$y(t) = x(t) * s(t)$$

$$AK\Phi[y(t)] = AK\Phi[x(t)] * AK\Phi[s(t)]$$



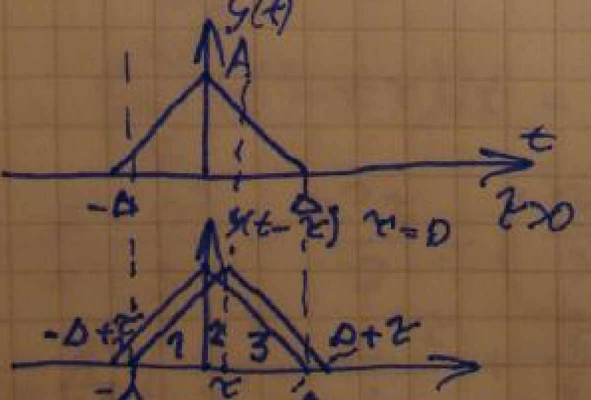


-2Δ
2Δ
t
$\delta(t-\tau)$
$\tau=0$
$R'(\tau)$
"3"
-4Δ
-2Δ
2Δ
4Δ
$\tau=2\Delta$
"2"
$\tau=4\Delta$
"1"

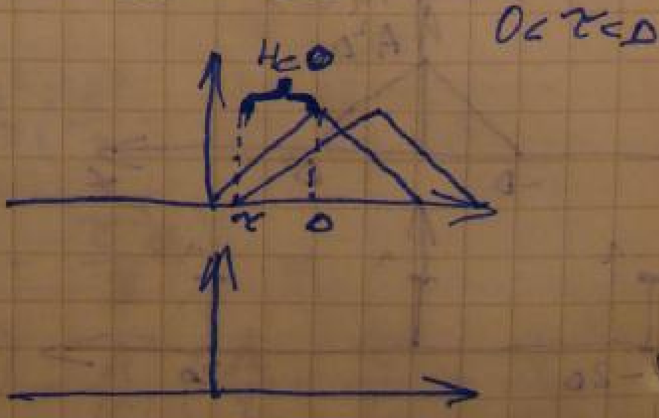


Rd(t)
(3)
(2)
(2)
(1)
(1)
-4Δ
-2Δ
2Δ
4Δ

$3A^2\Delta$
$2A^2\Delta$
$A^2\Delta$
$2A^2\Delta$
$A^2\Delta$
-5Δ
5Δ



y(t)
A
t
-Δ
Δ
$\tau>0$
$y(t-\tau)$
$\tau=\Delta$
$-\Delta+\tau$
1
2
3
$\Delta+\tau$
-Δ
τ
Δ



$0<\tau<\Delta$
$t<0$
τ
Δ

$$y(t) = \begin{cases} \frac{A}{\Delta}(t+\Delta), & -\Delta < t < 0 \\ -\frac{A}{\Delta}(t-\Delta) & 0 < t < \Delta \\ 0, \text{ ост. } t \end{cases}$$

$\cancel{0<\tau<\Delta} : R_1(\tau)$

$$R_1(\tau) = \int_{-\Delta+\tau}^{0} \frac{A}{\Delta}(t+\Delta)\frac{A}{\Delta}(t-\tau+\Delta)\,dt +$$

$$+\int_{0}^{\tau} -\frac{A}{\Delta}(t-\Delta)\cdot\frac{A}{\Delta}(t-\tau+\Delta)\,dt + \int_{\tau}^{\Delta} -\frac{A}{\Delta}(t-\Delta)\cdot$$

$$\cdot\left(-\frac{A}{\Delta}(t-\tau-\Delta)\right)dt$$

$\Delta < \tau < 2\Delta$

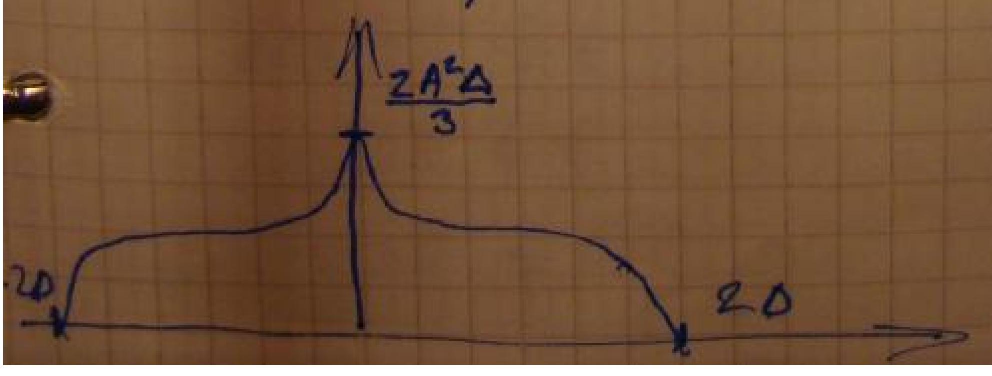
$$R_2(\tau) = \int_{\tau}^{\Delta} -\frac{A}{\Delta}(t-\Delta)\left(\frac{A}{\Delta}(t-\tau+\Delta)\right)dt$$

$\tau > 2\Delta$

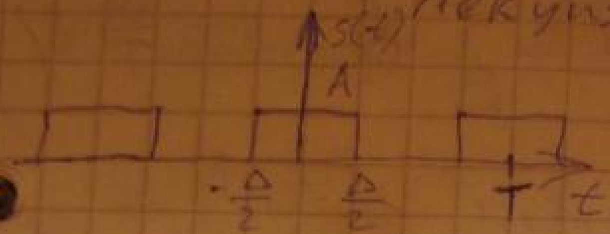
$$R_3(\tau) = 0$$

$-\infty < \tau < +\infty$

$$R(\tau) = \begin{cases} R_1(|\tau|), & 0 \le |\tau| \le \Delta \\ R_2(|\tau|), & \Delta \le |\tau| < 2\Delta \\ 0, & |\tau| \ge 2\Delta \end{cases}$$



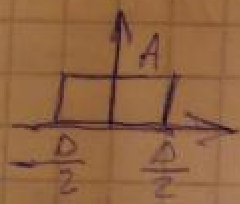
Лекция 4 02.10



T, A, τ

Q [скважность] $= \frac{T}{\tau}$

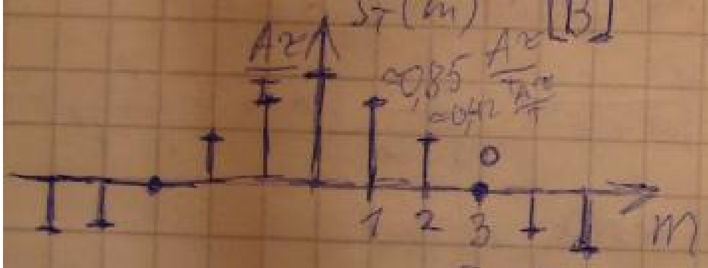
$S_T(m) = \frac{1}{T} S_1\left(\frac{m}{T}\right)$



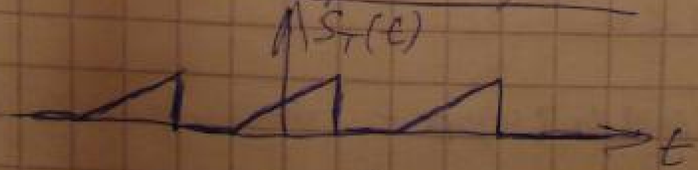
$S_1(f) = \text{sinc}(\pi f \Delta)\, A\tau$

$\Downarrow$

$S_T(m) = (A\tau/T)\,\text{sinc}\left(\pi \frac{m}{T} \tau\right)$ [В]



Ряд Фурье



$\cos 2\pi \frac{1}{T} t$ и $\sin 2\pi \frac{1}{T} t$ через $\frac{1}{T}$ инт.

$\cos 2\pi \frac{2}{T} t$ $\sin 2\pi \frac{2}{T} t$

$\cos 2\pi \frac{3}{T} t$ $\sin 2\pi \frac{3}{T} t$

C – можно считать, если $m = 0$

$$S_T(t) = \sum_{m=0}^{\infty} a_m \cos(2\pi \frac{m}{T} t) + \sum_{m=1}^{\infty} b_m \sin(2\pi \frac{m}{T} t)$$

$$a_0 = \frac{1}{T}\int_T S_T(t)\,dt = \frac{1}{T}\int_0^T S_T(t)\,dt = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}}$$

$m>0$ $$a_m = \frac{2}{T}\int_T s(t)\cos(2\pi \frac{m}{T} t)\,dt$$

$$b_m = \frac{2}{T}\int_T s(t)\sin(2\pi \frac{m}{T} t)\,dt$$

a_0 - постоянная составляющая

a_m - косинусный (синфазный) коэф.

b_m - синусный (квадратурный) коэф.

$$a\cos x + b\sin x = \sqrt{a^2+b^2}\left(\frac{a}{\sqrt{a^2+b^2}}\cos x + \frac{b}{\sqrt{a^2+b^2}}\sin x\right)$$

$$= \sqrt{a^2+b^2}\cos(x - \psi) = A\cos(x + \ )$$

$A = \sqrt{a^2+b^2}$ $\varphi = -\psi$

$$\psi = \begin{cases} \cos\psi = \frac{a}{\sqrt{a^2+b^2}} \\ \sin\psi = \frac{b}{\sqrt{a^2+b^2}} \end{cases} \qquad tg\,\psi = \frac{b}{a}$$

$$\varphi = \begin{cases} -arctg\frac{b}{a} & a>0 \\ \pi - arctg\frac{b}{a} & a<0 \end{cases}$$

$[A_m, \varphi_m]$ - амплитудно-фазовый РФ

$a_m = A_m\cos\varphi_m$

$b_m = -A_m\sin\varphi_m$

$$A\cos(x+\varphi) = \frac{A\left(e^{j(x+\varphi)} + e^{-j(x+\varphi)}\right)}{2} =$$

$$= \underbrace{\frac{A}{2}e^{j\varphi}}_{C} e^{jx} + \underbrace{\frac{A}{2}e^{-j\varphi}}_{C^*} e^{-jx} = C e^{jx} + C^* e^{-jx}$$

$$S_T(t) = \frac{A_0}{C_0} + \sum_{m=1}^{\infty} \left\{ C_m e^{j2\pi\frac{m}{T}t} + C_m^* e^{-j2\pi\frac{m}{T}t} \right\} =$$

$$= \sum_{m=-\infty}^{\infty} C_m e^{+j2\pi\frac{m}{T}t}$$

компл. ряд Фурье

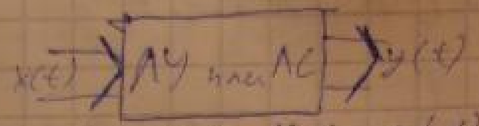
$$C_m = \begin{cases} A_0, & m = 0 \\ A_m e^{j\varphi_m} & m > 0 \\ A_{|m|} e^{-j\varphi_m} & m < 0 \end{cases}$$

$$C_m = S_T[m]$$

$$x(t) = e^{j2\pi F t} \Leftarrow\Rightarrow X(f) = \delta(f - F)$$

$$S_T(f) = \sum_{m=-\infty}^{\infty} S_T[m]\, \delta\left[f - \frac{m}{T}\right]$$

Прохождение сигналов через фильтры (лин устр)



ЧХ $H(f)$ $y(f)$

$X(f)$ $y(p)$

ИХ $h(t)$ реакц на δ-функ

$X(p)$

СФ $H(p)$

ДУ

$$Y(f) = X(f)\,H(f)$$

$$Y(p) = X(p)\,H(p)$$

$$y(t) = x(t) \cdot h(t)$$

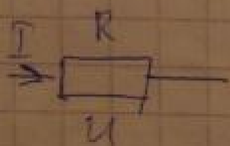
$X(f) = F[x(t)]$ – преобр Фурье

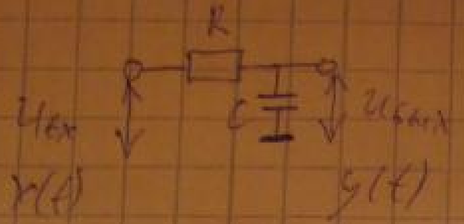
$X(p) = L\{x(t)\}$ – преобр Лапласа

$$X(f) = X(p)\big|_{p=2\pi f}$$

Не каждая система мож. быть описана с исп. этих хар

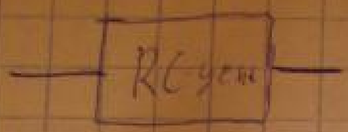
Линейные системы (фильтры)

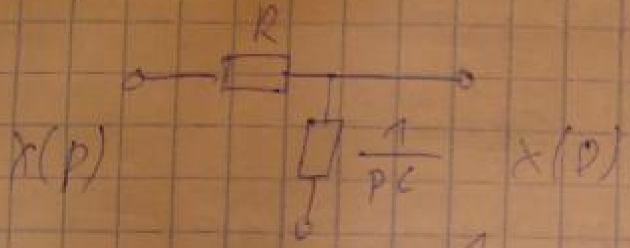




$\angle$ ФНЧ

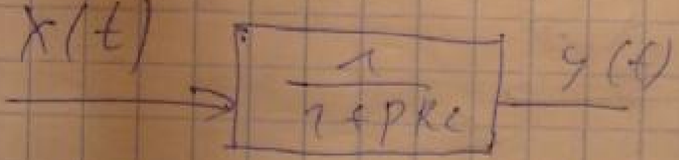
low pass filter





$$Y(p) = X(p)\,\frac{\frac{1}{pC}}{\frac{1}{pC} + R}$$

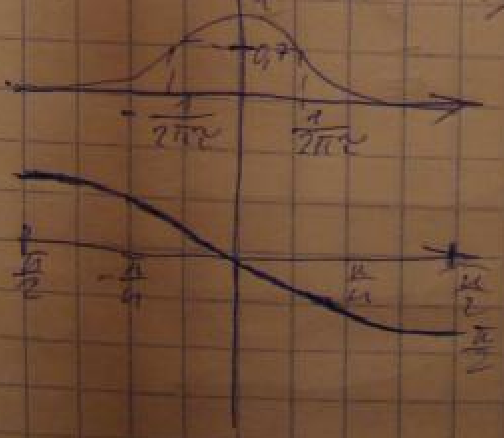
$$H(p) = \frac{Y(p)}{X(p)} = \frac{1}{1 + pRC}$$



$$H(f) = \frac{1}{1 + RC2\pi f}$$

$$АЧХ = \frac{1}{\sqrt{1 + (2\pi f\tau)^2}}$$

$$ФЧХ = -\operatorname{arctg} 2\pi f\tau$$



$$h(t) = L^{-1}\{H(p)\}$$

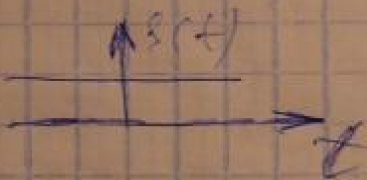
$$H(p) = \frac{1/\tau}{p + 1/\tau}$$

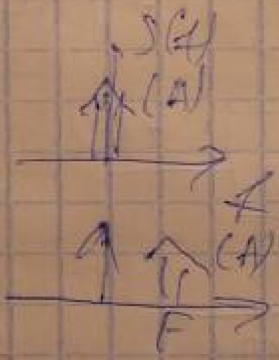
$$h(t) = \frac{1}{\tau} e^{-\frac{1}{\tau}t} u(t)$$

Свойство каузальности

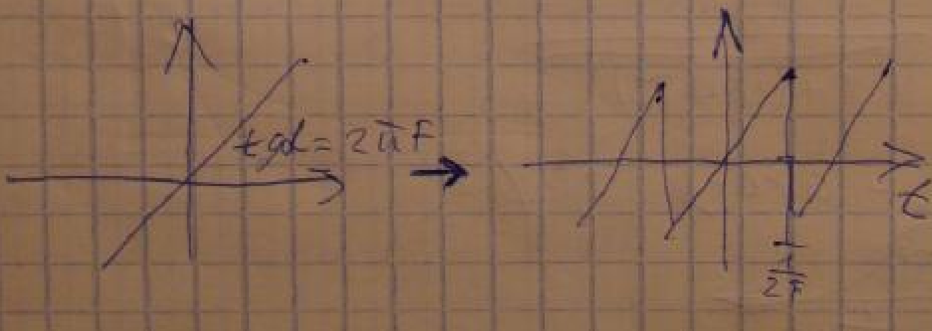
Все реальные устройства имеют каузальную имп. хар.

Идеальный ФНЧ





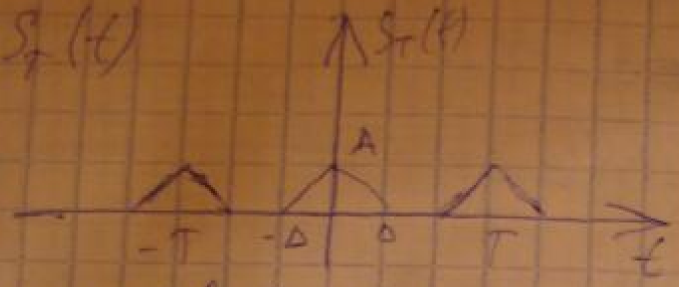
$A e^{j2\pi F t}$

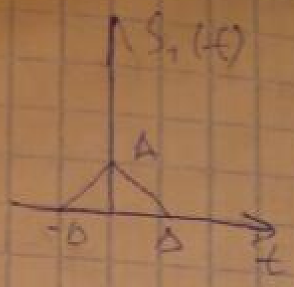


Периодические сигналы

Семинар 5

06.10



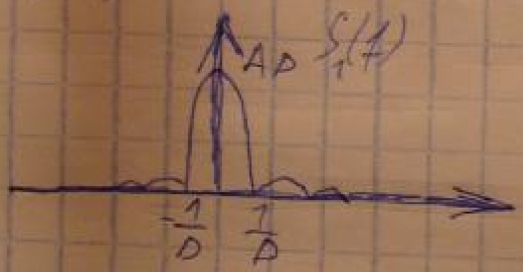


$$S_1(t)=\begin{cases}\frac{A}{\Delta}(t+\Delta), & -\Delta<t<0\\ -\frac{A}{\Delta}(t-\Delta) & 0<t<\Delta\end{cases}$$

$$S_T(t)=\sum_{k=-\infty}^{+\infty} S_1(t-kT)$$

$$S_T(f)=\sum_{m=-\infty}^{+\infty} S_T[m]\,\delta\left(f-\frac{m}{T}\right)$$

$$S_1(f)=A\Delta\,\mathrm{sinc}^2(\pi f\Delta)$$



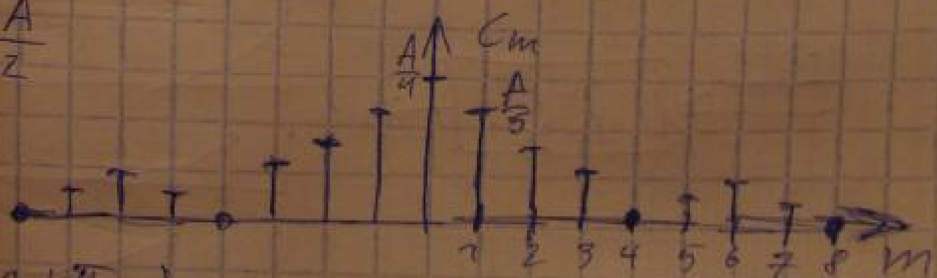
$$S_T[m]=C_m=\frac{1}{T}S_1\left[\frac{m}{T}\right]$$

$$S_T[m]=\frac{A\Delta}{T}\mathrm{sinc}^2(\pi f\Delta)$$

$T=2\Delta$

$S_T[0]=\frac{A}{2}$

$T=4\Delta$



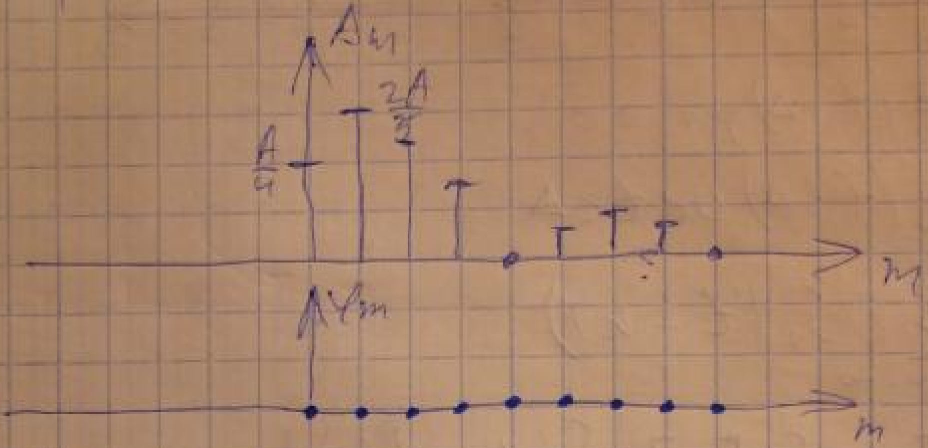
$$C_m=\frac{A}{4}\mathrm{sinc}^2\left(\frac{\pi}{4}m\right)$$

c_m – компл $\Rightarrow$ $|c_m|$ $\arg(c_m)$

$A_m = 2|c_m|, m>0$

$A_m = |c_m|; m = 0$

$\varphi_m = \arg c_m, m \geq 0$

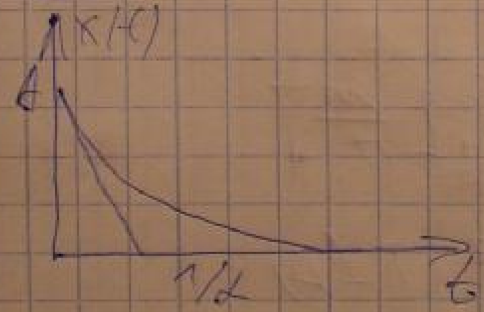


Квадратурная форма

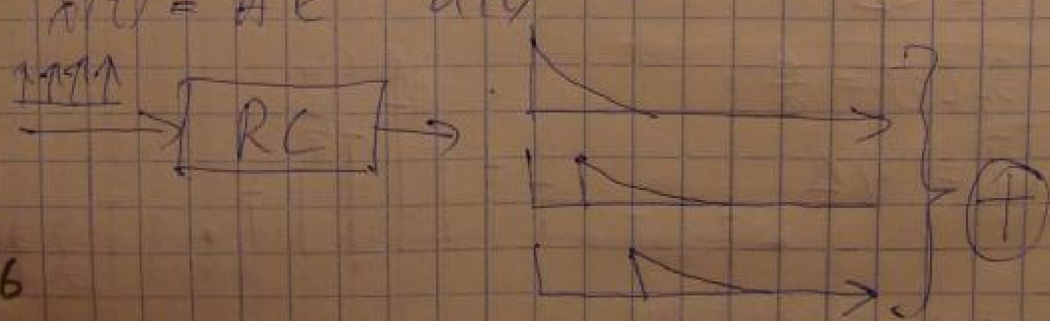
(a_m, b_m)

$a_m = A_m \cos\varphi_m$

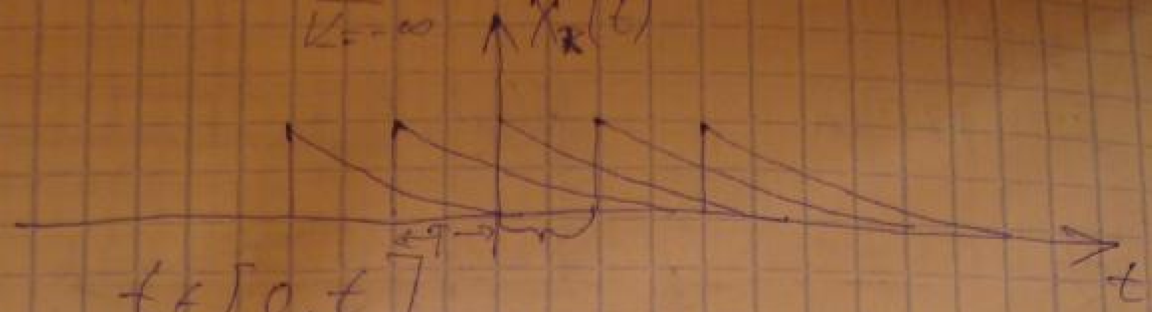
$b_m = -A_m \sin\varphi_m$



$x(t) = Ae^{-\alpha t}u(t)$



$$x_T(t) = \sum_{k=-\infty}^{+\infty} x(t-kT)$$



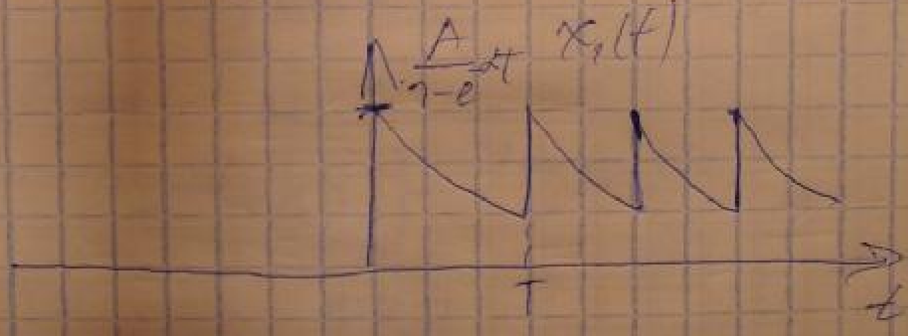
$$t \in [0, t]$$

$$Ae^{-\alpha t} + Ae^{-\alpha T}e^{-\alpha t} + Ae^{-2\alpha T}e^{-\alpha t} + Ae^{-3\alpha T}e^{-\alpha t}$$

$$b_0 = 1; \quad q = e^{-\alpha t} \qquad \underbrace{Ae^{-\alpha t}\left(1 + e^{-\alpha T} + e^{-2\alpha T} + \dots\right)}_{(*)}$$

$$(*) = Ae^{-\alpha t}\frac{1}{1-e^{-\alpha T}}$$

$$x_T(t) = \begin{cases} e^{-\alpha t}\dfrac{A}{1-e^{-\alpha T}} & 0 \le t < T \\ 0, & \text{ост.} \end{cases}$$



$$T = \frac{1}{\alpha}$$

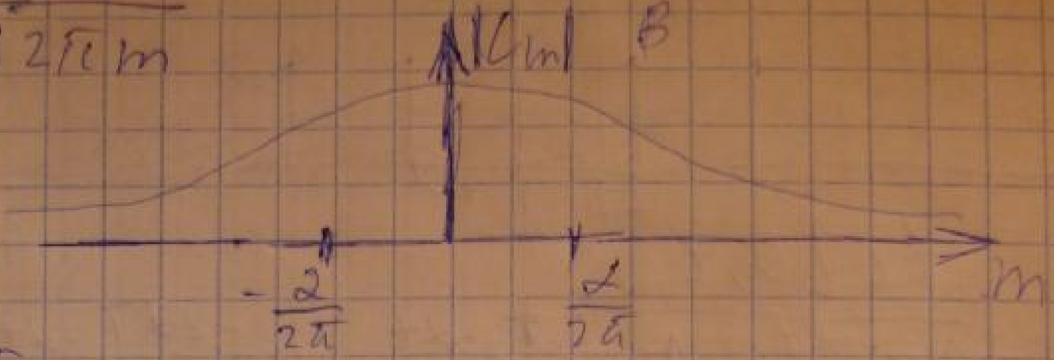
$$\frac{A}{1-e^{-\alpha t}} = \frac{A}{1-0{,}37} = \frac{A}{0{,}63} = 1{,}6A$$

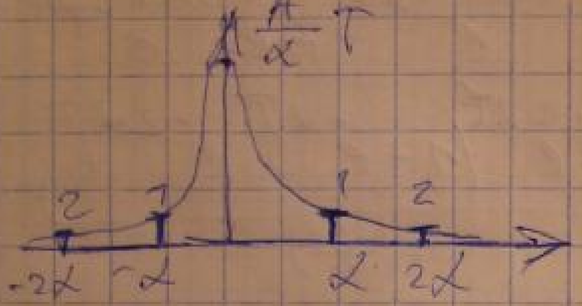
$$X(f) = \frac{A}{\alpha + j2\pi f}$$

$$X_T[m] = c_m = \frac{1}{T}\,\frac{A}{\alpha + j2\pi\frac{m}{T}} =$$

$$= \frac{A}{\alpha T + j2\pi m}$$

$$T = \frac{1}{\alpha}$$





m

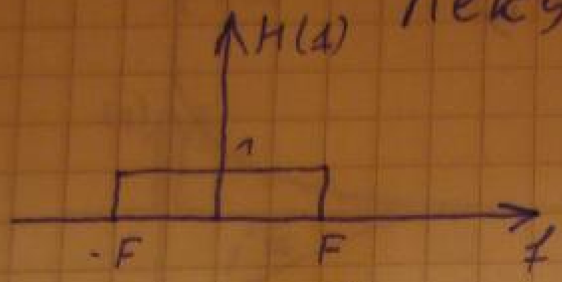
A

$$A_m = \frac{A\cdot 2}{|\alpha T + j2\pi m|}$$

$$\varphi_m = -\arg(\alpha T + j2\pi m)$$

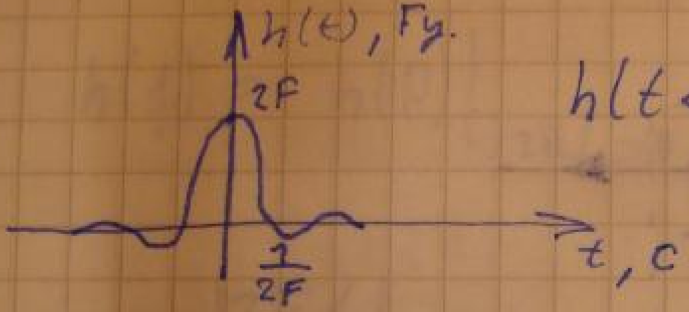
Лекция 5

09.10



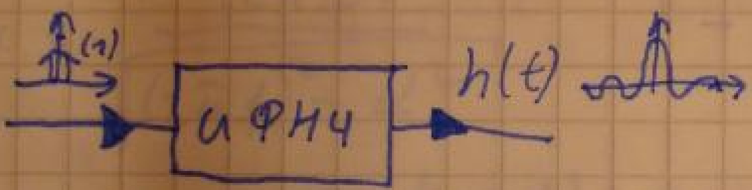
$$h(t) = F^{-1}[H(f)]$$

$$h(t) = 2F \operatorname{sinc}(2\pi\, 2F t)$$

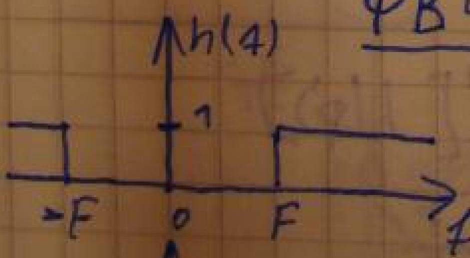


$h(t<0) \neq 0$ - не каузальна $\Rightarrow$

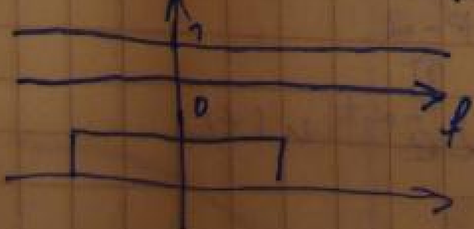
$h(p)$ - не сущ.



ФВЧ

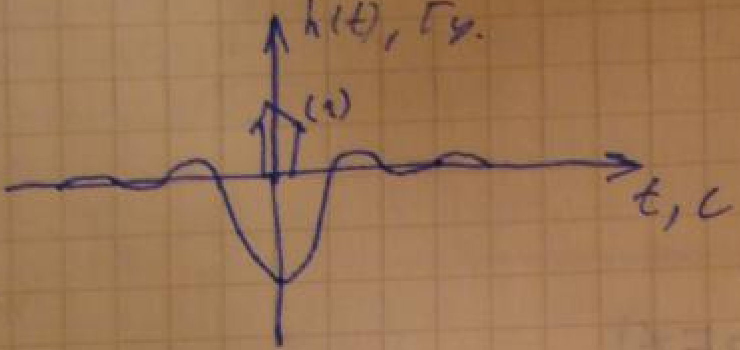


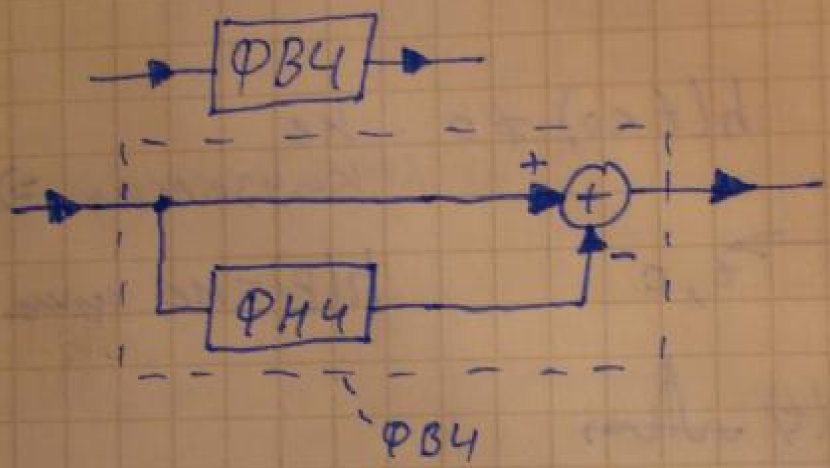
hi pass filter



ВПФ

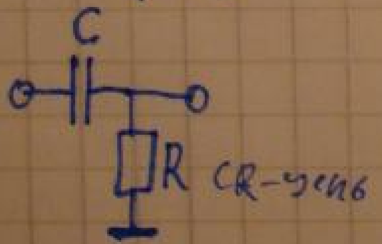
$$h(t) = \delta(t) - 2F\,\mathrm{sinc}(2\pi 2Ft)$$





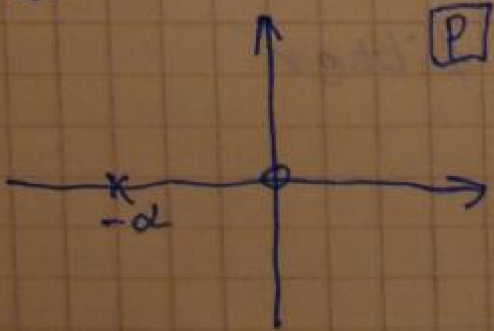
ФВЧ I

$$h(p) = \frac{p}{p-s}$$



$$= \frac{R}{R - \frac{1}{pC}} = \frac{p}{p+\frac{1}{RC}} = \frac{p}{p+\alpha}$$

днп

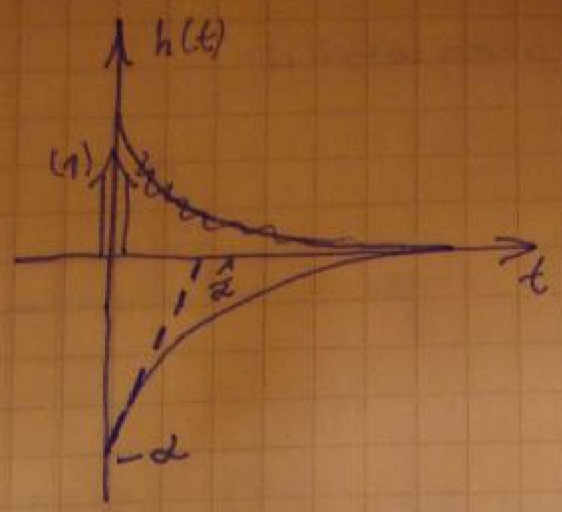


$\boxed{p}$ $$h(t) = L^{-1}[h(p)]$$

$$\frac{p}{p+\alpha} = 1 - \frac{p}{p-\alpha}$$

$$\Updownarrow \qquad \Updownarrow$$

$$\delta(t) - \alpha e^{-\alpha t} u(t)$$

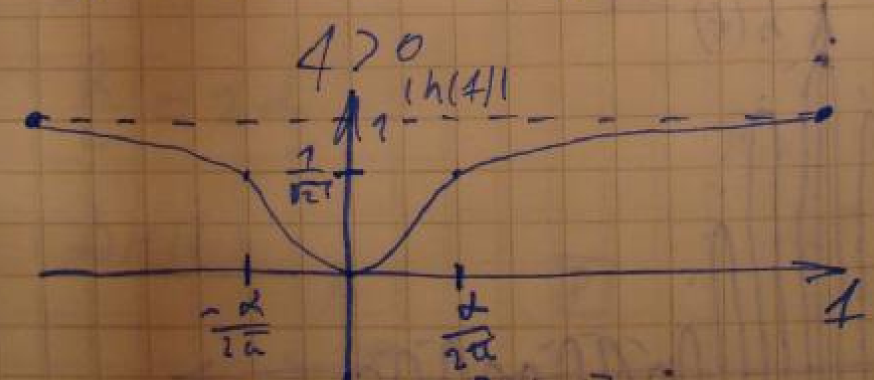


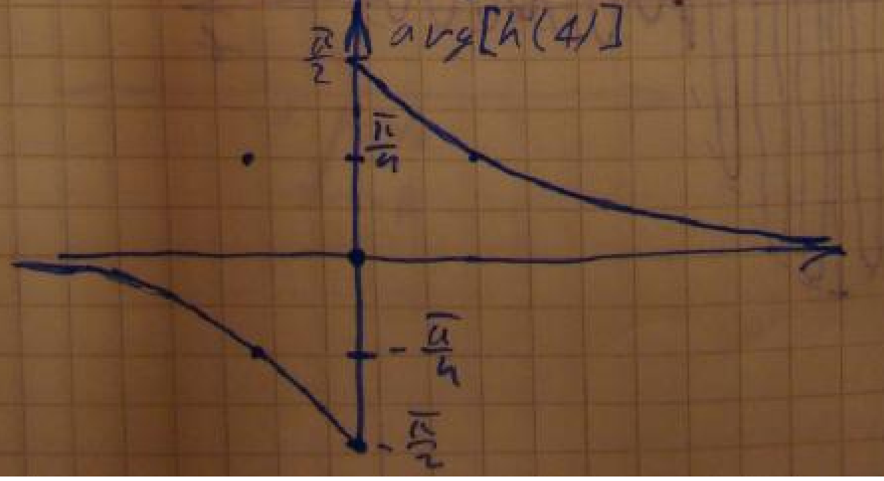
Появления $\delta(t)$ в ИХ характерно для фильтров, пропускающих ВЧ

$$h(f) = h(p)\Big|_{j2\pi f = p} = \frac{j2\pi f}{j2\pi f + \alpha}$$

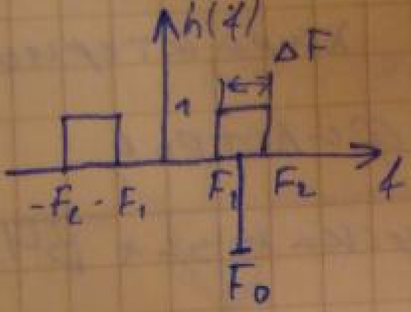
$$А ЧХ = \frac{|2\pi f|}{\sqrt{(2\pi f)^2 + \alpha^2}}$$

$$Ф ЧХ = \begin{cases} \arg(j2\pi f) - \arg(j2\pi f + \alpha) = \\ = \frac{\pi}{2} - \operatorname{arctg}\frac{2\pi f}{\alpha}, & f > 0 \\ -\frac{\pi}{2} - \operatorname{arctg}\frac{2\pi f}{\alpha}, & f < 0 \end{cases}$$



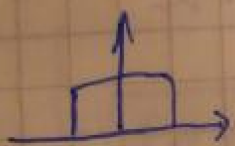


Полосо-пропускающий фильтр



$$F_0 = \frac{F_1 + F_2}{2}$$

$$\Delta f = F_2 - F_1$$

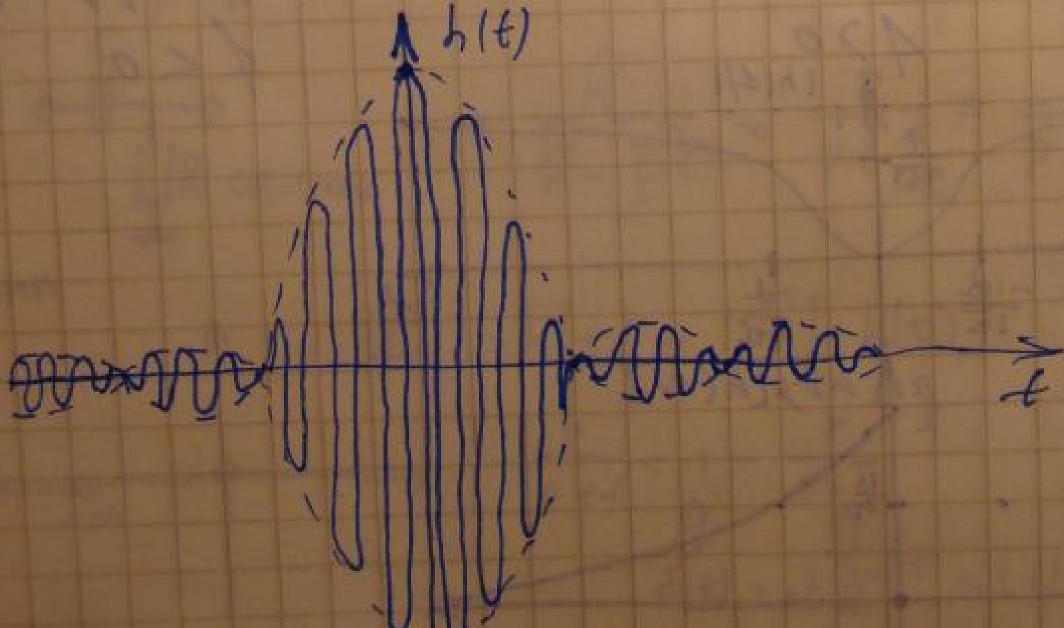


$$\Rightarrow \quad \Delta f \,\mathrm{sinc}(\pi \Delta f t)$$

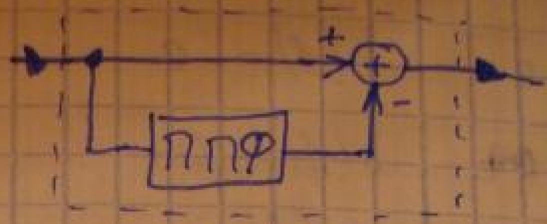
$$h(t) = \Delta F \,\mathrm{sinc}(\pi \Delta f t)\left(e^{+j2\pi F_0 t} + e^{-j2\pi F_0 t}\right) =$$

$$= \Delta f \,\mathrm{sinc}(\pi \Delta f t) \cos(2\pi f_0 t)$$

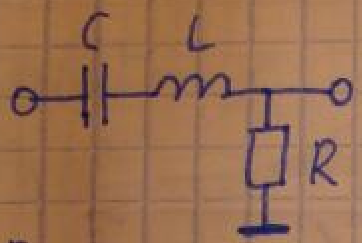
$$\frac{1}{\Delta f} \qquad \frac{1}{F_0}$$



Полосно-задерживающий фильтр



$$h(t) = \delta(t) - 2\Delta F \operatorname{sinc}(\pi \Delta F t) \cos(2\pi F_0 t)$$



$$h(p) = \frac{pRC}{p^2 LC + pRC + 1} =$$

$$= \frac{\frac{R}{L} p}{p^2 + \frac{R}{L} p + \frac{1}{LC}} = \frac{2\alpha p}{p^2 + 2\alpha p + \omega_0^2}$$

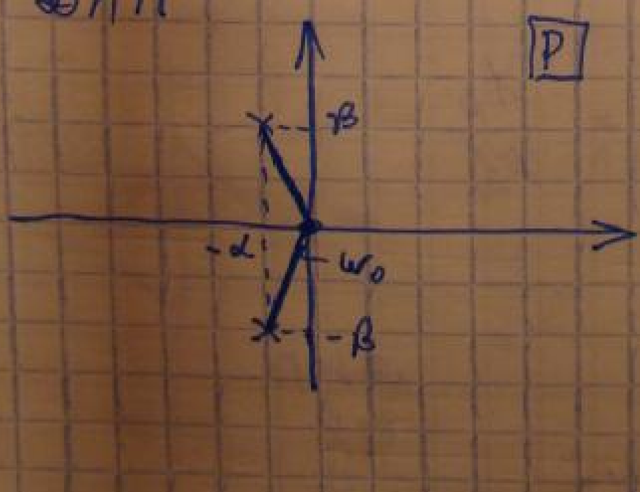
$$\frac{1}{LC} = \omega_0^2 \qquad Q = \frac{\omega_0}{2\alpha}$$

$$\frac{R}{L} = 2\alpha$$

$$p_{1,2} = \omega_0 \sqrt{1 - \frac{1}{4Q^2}}$$

$Q\uparrow \quad \beta \to \omega_0$

ДНП



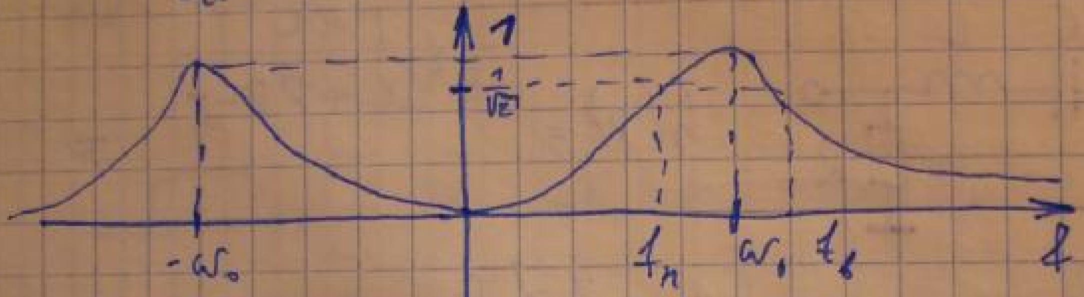
$$H(p)\Big|_{p=j2\pi f} = \frac{2\alpha\, j2\pi f}{-(2\pi f)^2 + \omega_0^2 + 2\alpha j 2\pi f}$$

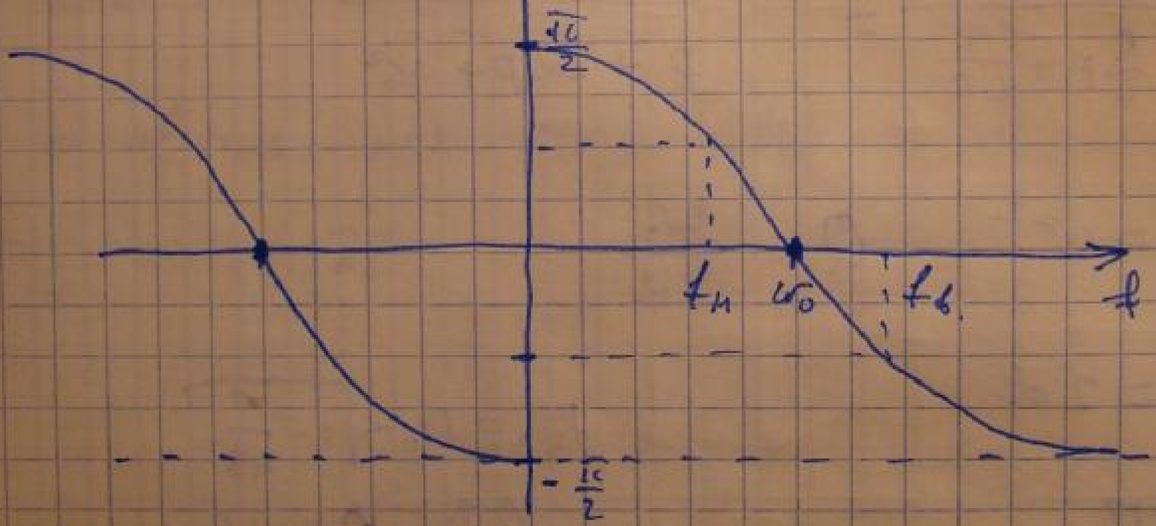
$f = 0 \quad h(f) = 0$

$f \to \cancel{0+} \quad h(f) = 0+$

$f \to 0- \quad H(f) = 0-$

$f = f_0 = \frac{\omega_0}{2\pi} \quad h(f) = 1$





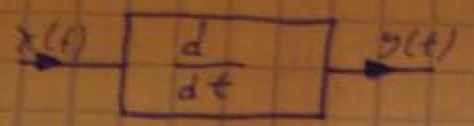
$f_0 = \sqrt{f_b f_n} \qquad Q = \frac{f_0}{\Delta f}$

UX $\quad H_p(f)$

$$F_{pm} = \frac{p^2 + \omega_0^2}{p^2 + 2\alpha p + \omega_0^2}$$

$\cancel{\Delta}$ ЧП
ЧХ
ЧХ

Дифференциальный фильтр

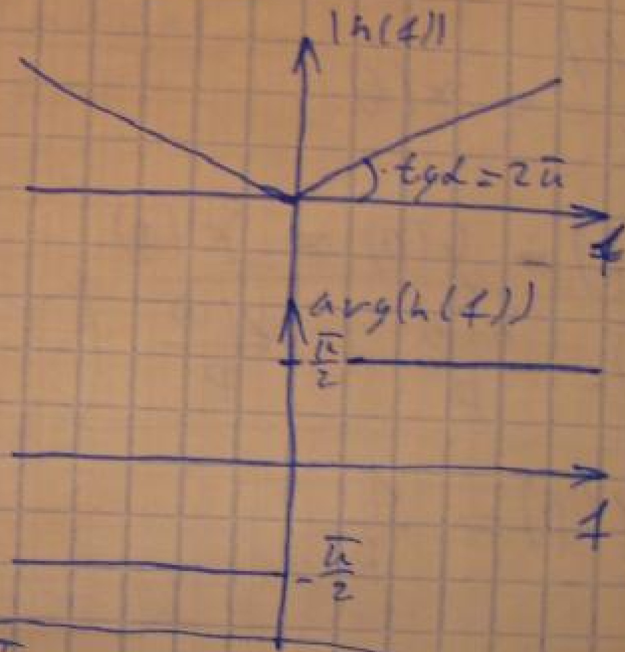


$$y(t) = \frac{dx(t)}{dt} \Big\} \text{Пл}$$

$$y(p) = p\,X(p)$$

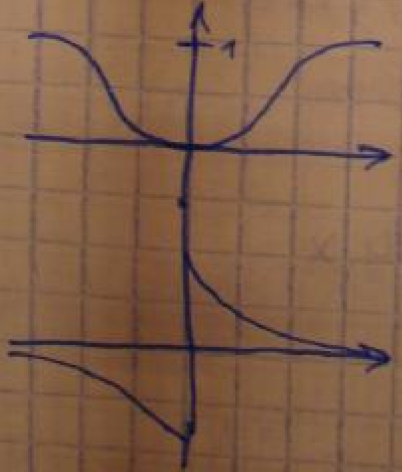
$$h(p) = p$$

$$h(f) = j2\pi f$$



Плохота вблизи 0 ⚠

ФВЧ I

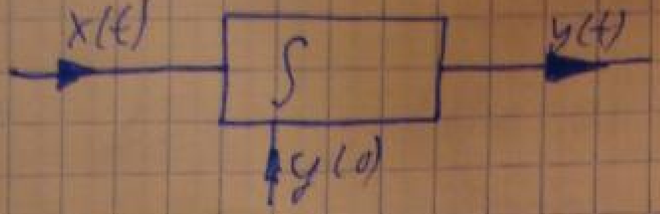


$$h(t) = \text{ОПЛ}\{p\}$$

$$\text{ОПЛ}\{1\} = \delta(t)$$

$$\frac{d}{dt}\delta(t) = \underline{\underline{\dot{\delta}(t) - \text{дублет}}}$$

Интегратор



$$y(t) = \int_{-\infty}^{t} x(t)\,dt = \underbrace{\int_{-\infty}^{0} x(t)\,dt}_{y(0)} + \int_{0}^{t} x(t)\,dt$$

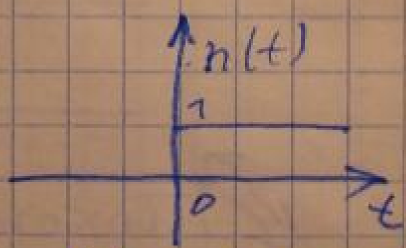
$$\frac{dy(t)}{dt} = x(t)$$

$$Y(p) = y(0) = X(p)$$

$$\boxed{X(p) = \frac{X(p)}{p} + \frac{y(0)}{p}}$$

$$H(p) = \frac{1}{p}$$

$$H(t) = u(t)$$



Не затухає

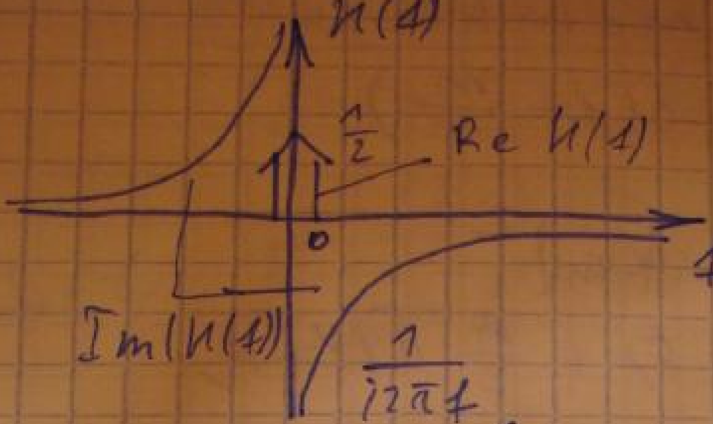
$$\int_{-\infty}^{\infty} u(t)\,dt \neq C < \infty$$

СФ → ЧХ → ЧХ

$$u(t) \Leftrightarrow \frac{1}{j2\pi f} + \frac{1}{2}\delta(f)$$

$$\Updownarrow$$

$$H(f)$$



Идеальный интегратор

похож на ФНЧ

Семинар 6 13.10

Мощность периодических сигналов

$$P_{ср} = \frac{1}{T}\underbrace{\int_T s^2(t)\,dt}_{Э_T}$$

$$s(t) = A\cos(2\pi F t)$$

$$Э_{ср} = \int_0^T A^2\cos^2(2\pi F t)\,dt$$

$$\cos(2\alpha) = \cos^2\alpha - \sin^2\alpha = 2\cos^2\alpha - 1$$

$$\cos^2\alpha = \frac{1}{2}\cos 2\alpha + \frac{1}{2}$$

$$= \int_0^T \left(A^2\frac{\cos(4\pi F t)}{2} + \frac{A^2}{2}\right)dt = \frac{A^2}{2}\int_0^T A^2\cos(4\pi F t$$

$$= A^2\left(\frac{\sin(4\pi F t)}{8\pi F}\Big|_0^T + \frac{t}{2}\Big|_0^T\right) = A^2\frac{T}{2}$$

$$P_{ср} = \frac{A^2}{2} = [В^2]$$

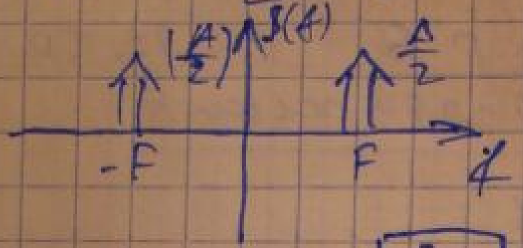
$$\boxed{P_{const\ ср} = (const)^2}$$

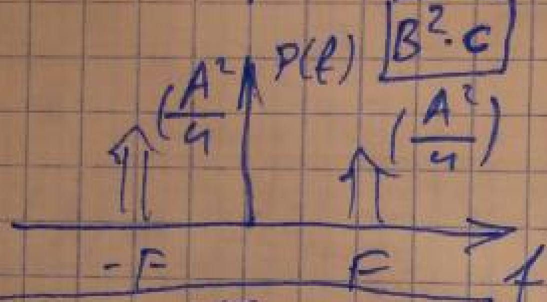
Спектр мощности

$$P(f) = \sum_{m=-\infty}^{+\infty} P_m \, \delta\left(f - \frac{m}{T}\right)$$

$$P_m = |C_m|^2$$

$$S(f) = \frac{A}{2}\delta(f+F) + \frac{A}{2}\delta(f-F)$$





$$\boxed{P_{cp} = \sum_{m=-\infty}^{+\infty} P_m}$$

$$P_{cp} = 2 \cdot \frac{A^2}{4} = \frac{A^2}{2}$$

$$P_{cp}\left(y^2(t) + x^2(t)\right) = P_{cp}\left(y^2(t)\right) + P_{cp}\left(x^2(t)\right)$$

Если спектра не пересекаются

Восстановление периодич. сигнала по части спектра

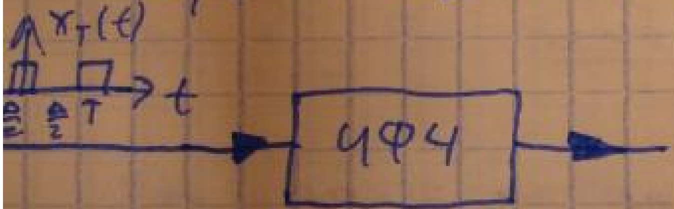
$$s_T(t) = \sum_{m=-\infty}^{+\infty} c_m e^{+j2\pi \frac{m}{T} t} =$$

$$c_m = S_T[m] = \frac{1}{T} S_1\left(\frac{m}{T}\right)$$

$$= A_0 + \sum_{m=1}^{\infty} A_m \cos\left(2\pi \frac{m}{T} t + \varphi_m\right)$$

$$\left.\begin{aligned} A_m &= 2|c_m| \\ A_0 &= c_0 \\ \varphi_m &= \arg c_m \end{aligned}\right\} m \neq 0$$

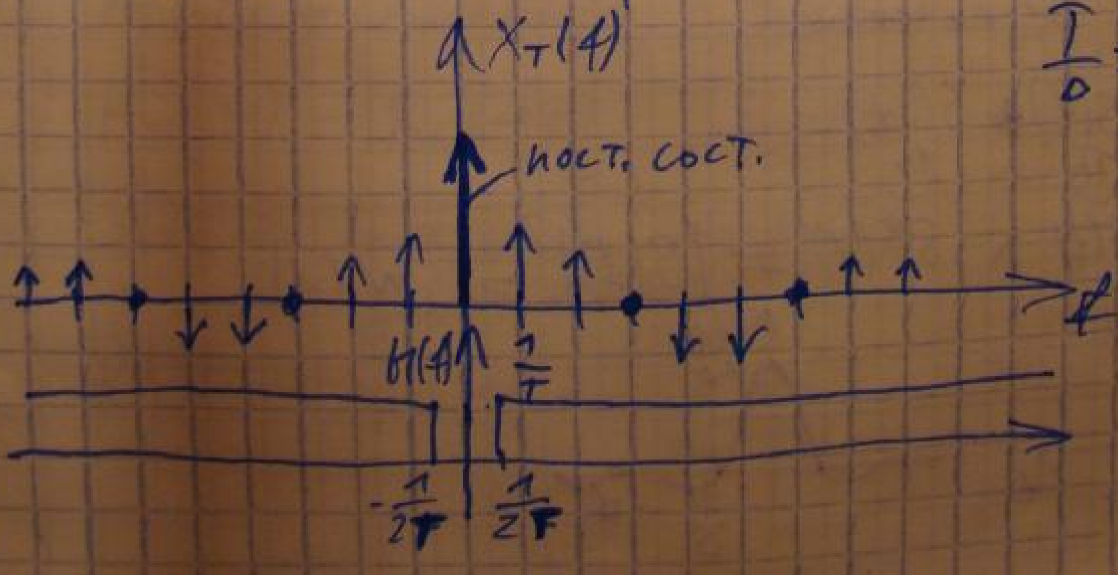
$$s_T(t) = s_= + s_\sim(t)$$



$$x_1(t) \Leftrightarrow X(f) = A\Delta\, \mathrm{sinc}(\pi f \Delta)$$

$$X_T[m] = c_m = \frac{A\Delta}{T} \mathrm{sinc}\left(\pi \frac{m}{T} \Delta\right)$$

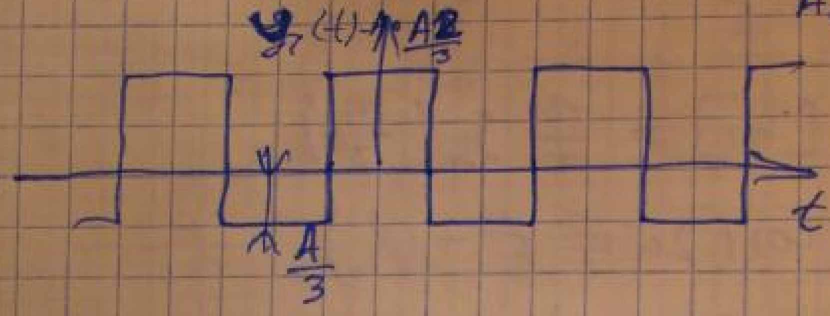
$$\frac{T}{\Delta} = 3$$



$$y_T(t) = \{c_0 = 0\} = \sum_{m=-\infty}^{+\infty} c_m e^{+j2\pi \frac{m}{T} t}$$

$$s_T(t) - A_0 = \sum_{m=1}^{\infty} A_m \cos\left(2\pi \frac{m}{T} t + \varphi_m\right)$$

$$A_0 = \frac{A}{3}$$



Искл. из сигнала первую гармонику

Лекция 6
23.10

Энергетические характеристики имп. и периодич. сигналов

$s(t) \overset{F}{\Leftrightarrow} S(f)$

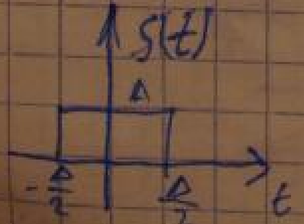
$R(\tau) \overset{F}{\Leftrightarrow} W(f)$

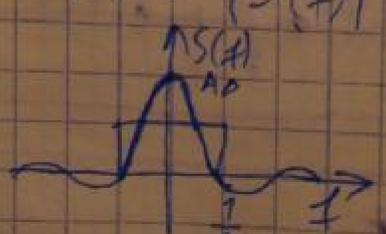
АКФ связана с энерг. спектром преобраз. Фурье, как сигнал со спектром

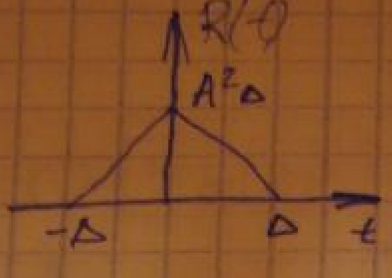
Теорема Винера-Хинчина

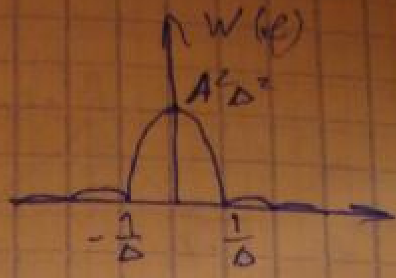
$$R(\tau) = s(t) * s(-t)$$

$$S(f) \times S^*(f) = |S(f)|^2 = W(f)$$

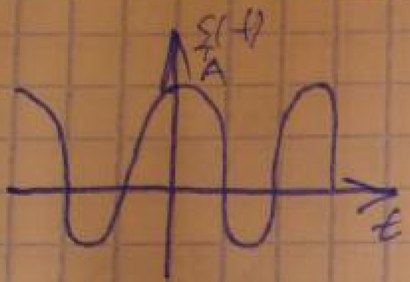








Мощностные хар. периодических сигналов



$$P_{ср} = \frac{A^2}{2}$$

$$P(f) = \sum_{m=-\infty}^{\infty} p_m \delta\left(f - \frac{m}{T}\right)$$

$$P_{ср} = \sum_{m=-\infty}^{+\infty} p_m$$

$$R_T(\tau) = \frac{1}{T}\int_T s_T(t)\cdot s_T(t-\tau)\,dt = [B^2]$$

$[B^2]$ – ед мощности

1) $R_T(0) = P_{ср}$

2) $R_T(\tau)$ – периодич, с периодом T

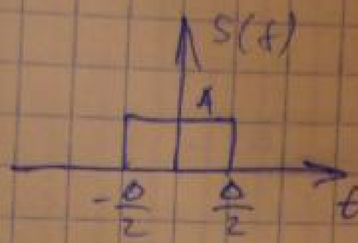
3) $R_T(\tau) = R_T(-\tau)$ $(Ev(\tau))$

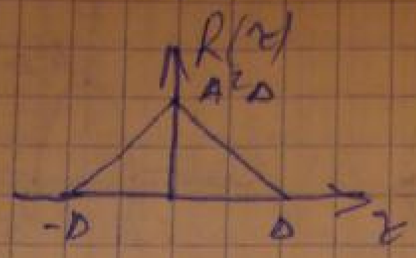
$$s(t) \Rightarrow s_T(t) = \sum_{m=-\infty}^{\infty} s(t-nT)$$

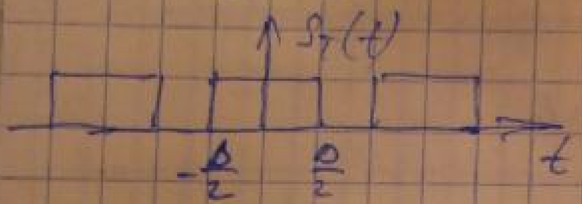
$$\Downarrow \qquad\qquad \Downarrow$$

$$R(\tau) \qquad R_T(\tau) = \frac{1}{T}\sum_{n=-\infty}^{\infty} R(\tau - nT)$$

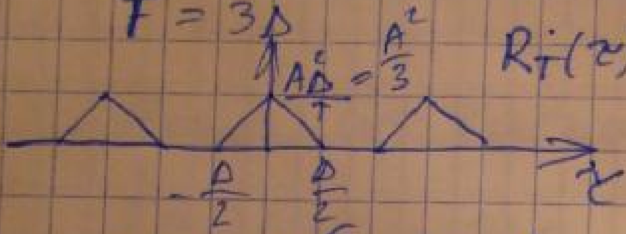
Пример:







$T = 3\Delta$



$R_T(\tau)$ [$В^2$]

$R_T(\tau) \Longleftrightarrow P(f)$ — част. случ. теоремы Винера-Хинчина

$$R_T(\tau) = \sum_{m=-\infty}^{\infty} P_m \, e^{+j 2\pi \frac{m}{T} t}$$

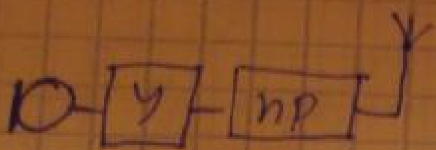
$$\begin{array}{ccc} s(t) & \Rightarrow & C_m \\ \downarrow & & \downarrow \\ P_T(t) & \Leftarrow & P_m \end{array}$$

Теорема Парсеваля

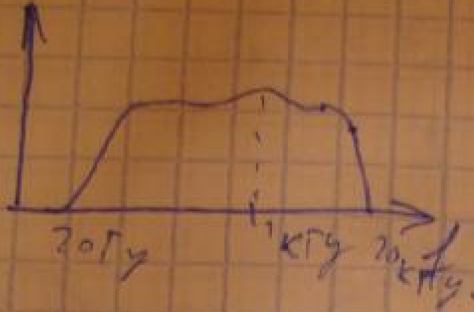
$$Э = \int_{-\infty}^{+\infty} |s(t)|^2 dt = \int_{-\infty}^{+\infty} |S(f)|^2 df$$

$$P_ф = \frac{1}{T} \int_T |s_T(t)|^2 dt = \sum_{m=-\infty}^{\infty} (c_m)^2$$

Теория модуляции

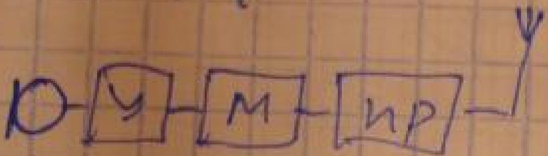


20Гц - 20кГц



$\frac{20000}{20} = 1000$ — широкополосный сигнал.

$c = \lambda f$



$13_{10} = 1101_2$

$s(t) = A\cos(2\pi F t + \varphi_0)$

$A \to A(t)$, $F \to F(t)$, $\varphi_0 \to \varphi_0(t)$

АМ ЧМ ФМ

$\text{(AM)} \quad \underbrace{\text{(FM)} \quad \text{(PM)}}_{\text{УМ - угловая мод.}}$

АМ+УМ - сложная мод.

Амплитудная мод.

Классическая АМ

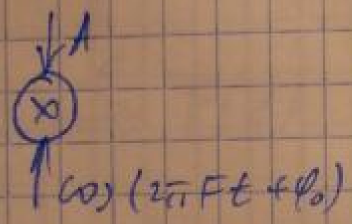
$x(t)$ – полезный сигнал.

$s(t)$ – несущий сигнал.

$A \to A(t)$

$$s(t) = A(t)\cos(2\pi F t + \varphi_0)$$

$$\boxed{A > 0}$$

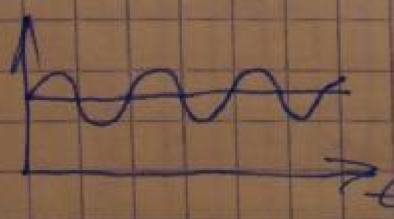


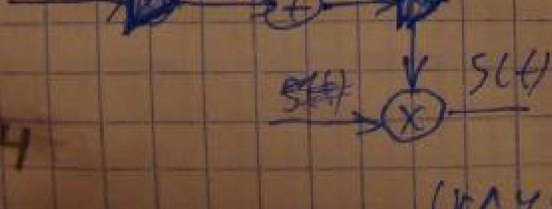
$$\mathrm{Min}[x(t)] = x_{min} < 0$$

$$\Rightarrow x(t) + (-x_{min}) = x_1(t)$$

$$x(t) = A\cos(2\pi F_x t) \quad - \quad (-A; A)$$

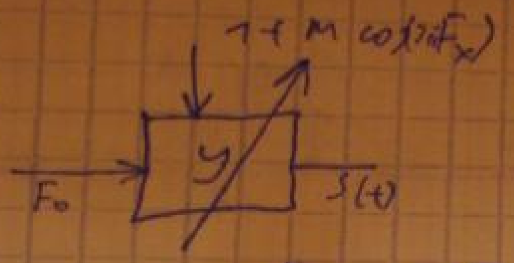
$$0 \leq [N x(t) + 1] < 1$$



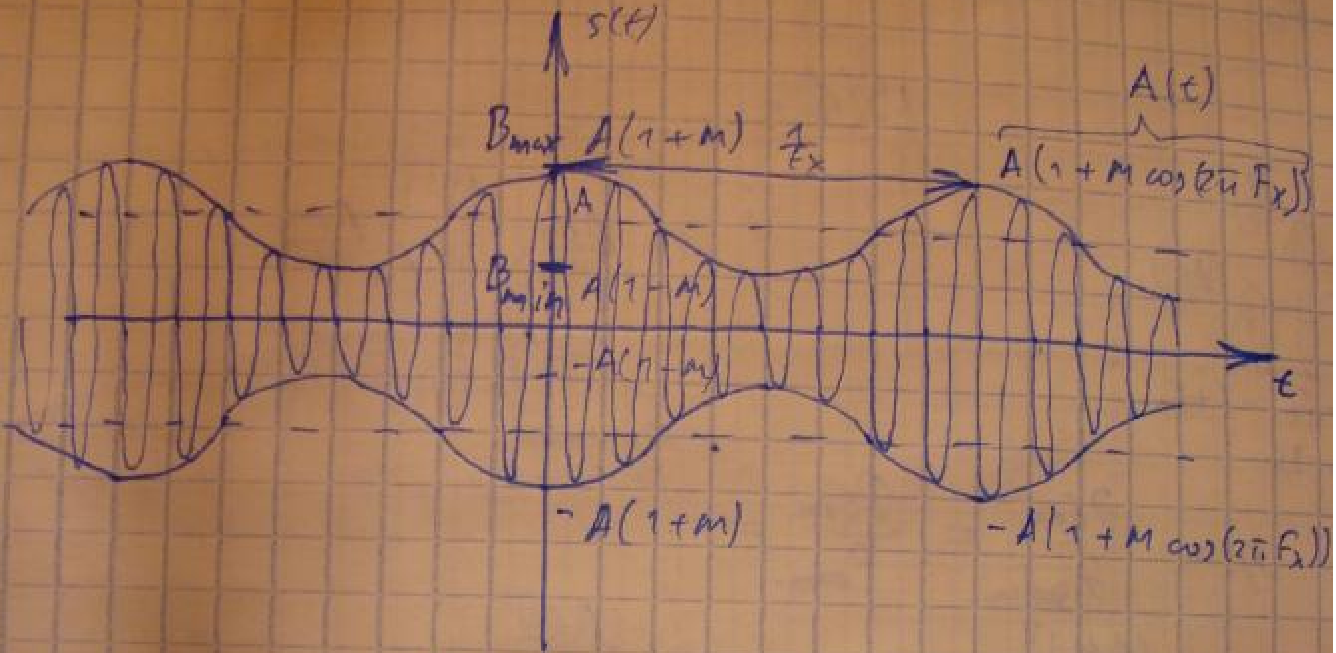


$$s(t) = A(1 + m\cos(2\pi F_x))\cos(2\pi F_0)$$

$0 < m < 1$ – коэф. ампл. модуляции (глубина АМ)



$$F_x << F_0$$



$$M = \frac{B_{max} - B_{min}}{B_{max} + B_{min}}$$

Спектр АМ-сигнала

$$s(t) = A\cos(2\pi F_0 t) + AM\cos(2\pi F_0 t)\cos(2\pi F_x t) =$$

$$= A\cos(2\pi F_0 t) + \frac{AM}{2}\cos(2\pi(F_0 + F_x)t) +$$

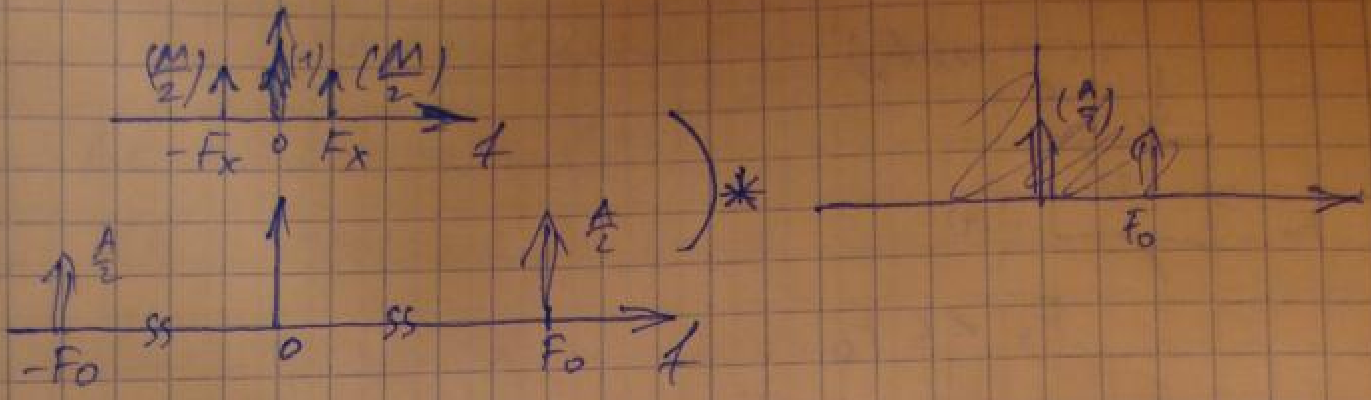
$$+ \frac{AM}{2}\cos(2\pi(F_0 - F_x)t)$$

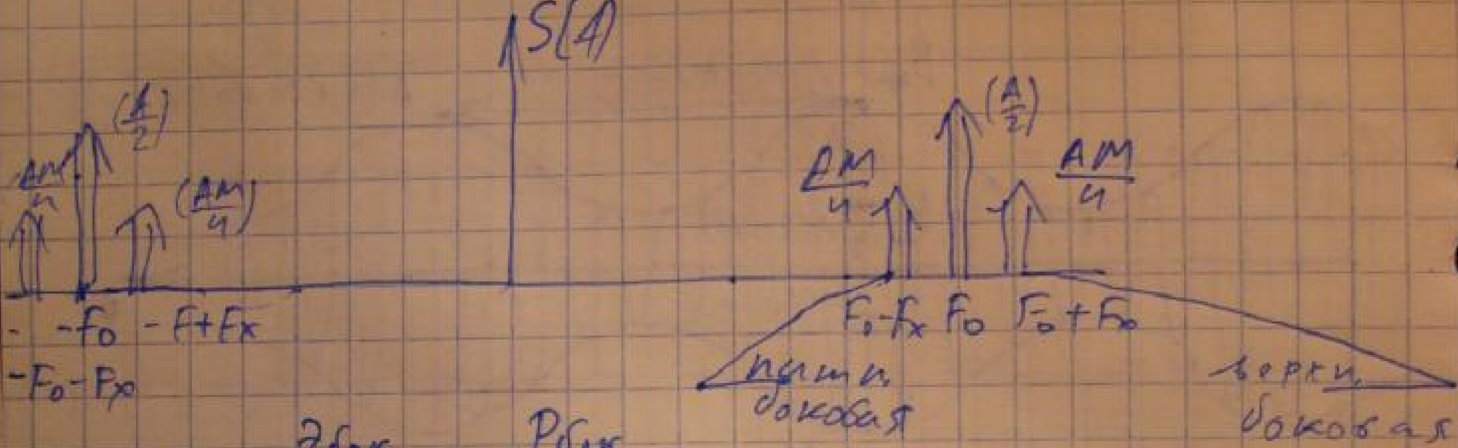
$$s(t) = [1 + M\cos(2\pi F_x t)]\, A\cos(2\pi F_0 t)$$

$$\Updownarrow$$

$$S(f)\ \left\{\delta(f) + \frac{M}{2}\delta(f - F_x) + \frac{M}{2}\delta(f + F_x)\right\} *$$

$$* \frac{A}{2}\delta(f - F_0) + \frac{A}{2}\delta(f + F_0)$$





$$\eta = \frac{Э_{бок}}{Э_{общ.}} = \frac{P_{бок}}{P_{общ.}} =$$

$$= \frac{4\cdot\left(\frac{AM}{4}\right)^2}{\left(4\cdot\frac{AM}{4}\right)^2 + 2\cdot\left(\frac{A}{2}\right)^2} = \frac{\frac{(AM)^2}{4}}{\frac{(AM)^2}{4} + \frac{A^2}{2}} =$$

$$= \frac{M^2}{M^2+2}$$

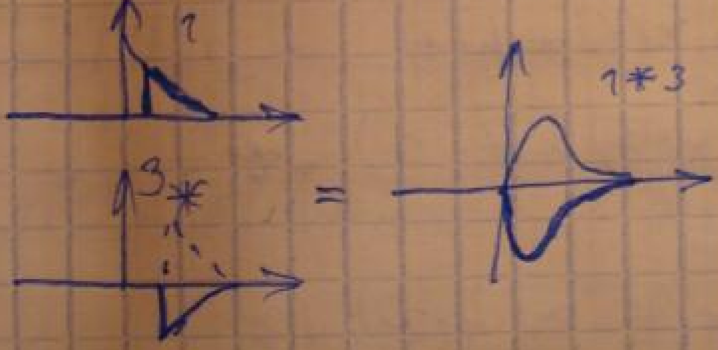
$\eta_{max} = \frac{1}{3}$ – низкая эффективность классической амп. мод.

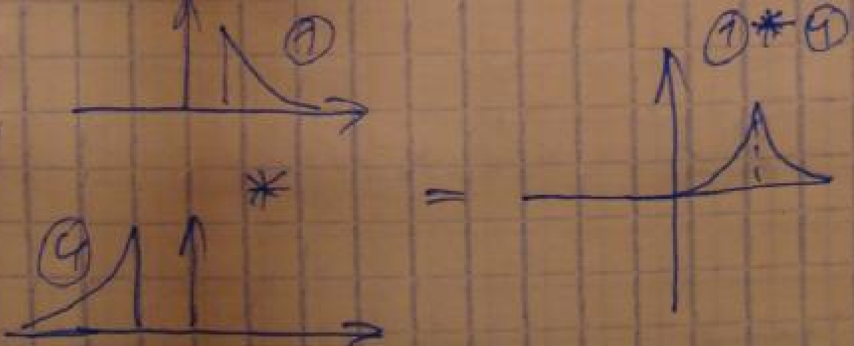
потеря информации.

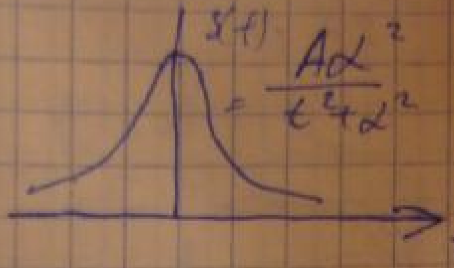
Семинар 27.10

$(1 + 2) * (3 + 4)$

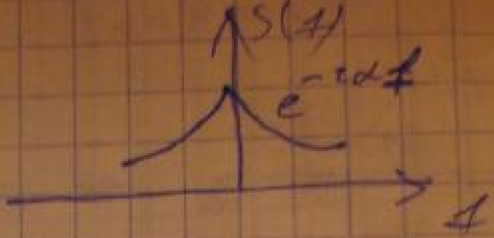
$1 * 3 + 1 * 4 + 2 * 3 + 2 * 4$

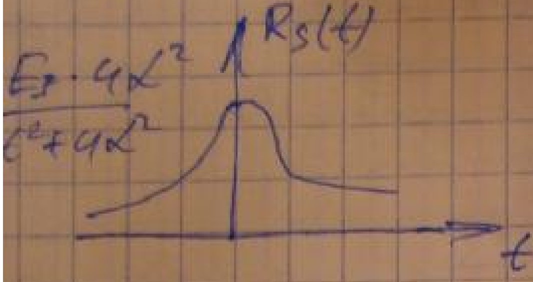


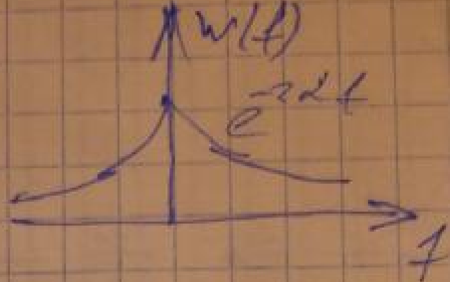




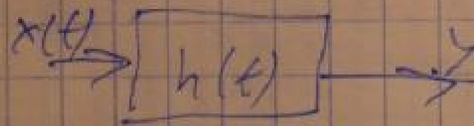
$\Longleftrightarrow$



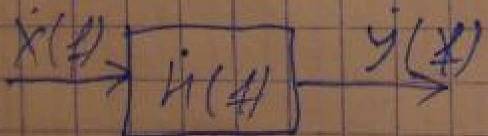




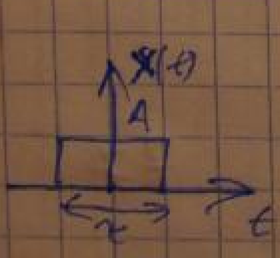
Прохождение имп. сигналов через линейные цепи



$$y(t) = x(t) * h(t) = \int_{-\infty}^{+\infty} x(\tau)\, h(t-\tau)\, d\tau = \int_{-\infty}^{+\infty} x(t-\tau)\, h(\tau)\, d\tau$$

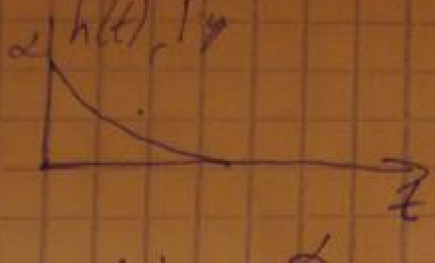


$$\dot{Y}(f) = \dot{X}(f) \times \dot{H}(f)$$

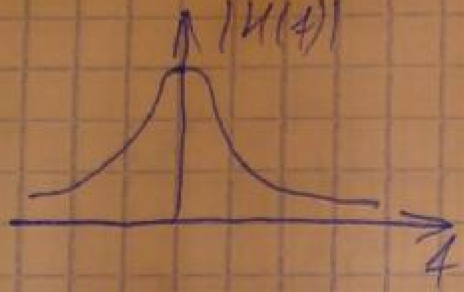


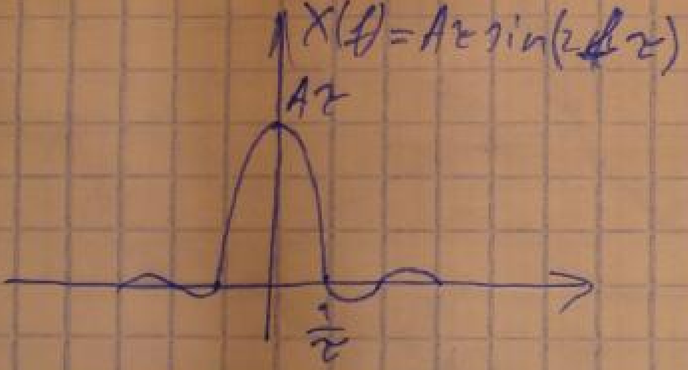
$$H(p) = \frac{\alpha}{p+\alpha} \xleftrightarrow{L^{-1}} h(t) = \alpha e^{-\alpha t} u(t)$$

$$\alpha = \frac{1}{\tau} = \frac{1}{RC}$$

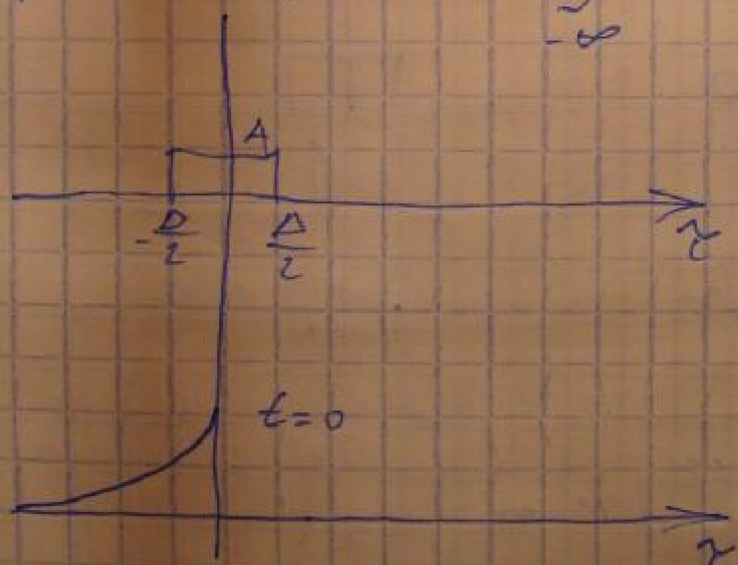


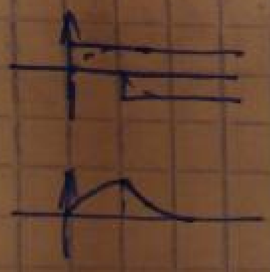
$$H(f)=\frac{\alpha}{j2\pi f+\alpha}$$

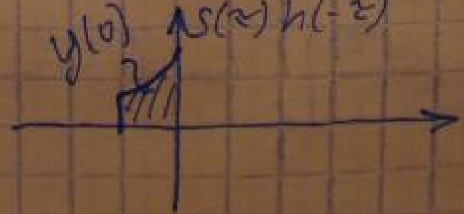


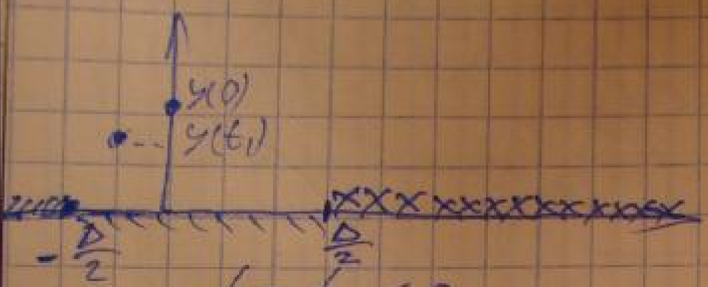


$$y(t)=x(t)*h(t)=\int_{-\infty}^{+\infty}x(\tau)\,h(t-\tau)\,d\tau$$







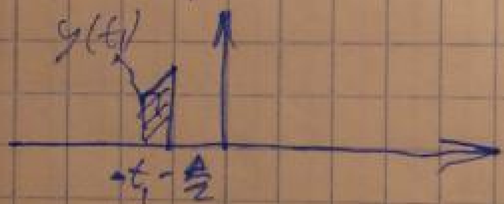


$t = t_1 < 0$

$h(t_1 - \tau)$

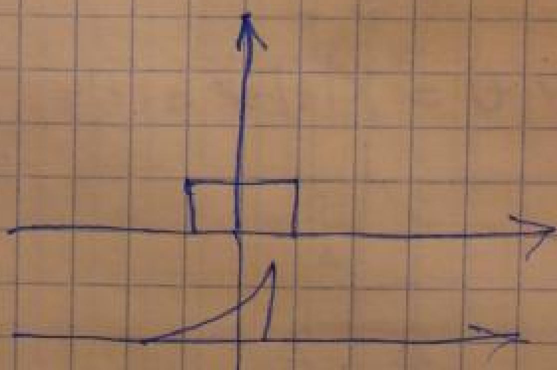
$t_1 - \tau = 0$

$\tau = t_1$



① $-\infty < t < -\frac{\Delta}{2}$

$$y(t) = \int_{-\infty}^{+\infty} 0 \, d\tau = 0$$



② $-\frac{\Delta}{2} < t < \frac{\Delta}{2}$

$$y(t) = \int_{-\frac{\Delta}{2}}^{t} A\alpha e^{-\alpha(t-\tau)} d\tau$$

$$y(t) = A\alpha e^{-\alpha t} \int_{-\frac{\Delta}{2}}^{t} e^{\alpha\tau} d\tau =$$

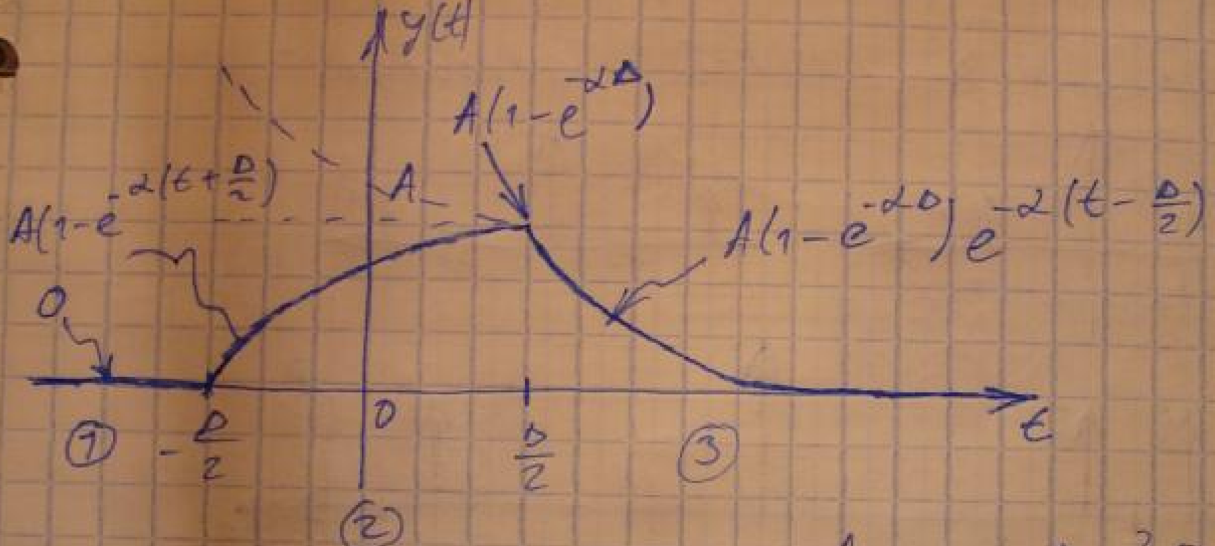
$$= A\alpha e^{-\alpha t} \left(\frac{1}{\alpha} e^{\alpha\tau} \Big|_{-\frac{\Delta}{2}}^{t} \right) = A\left(1 - e^{-\alpha(t+\frac{\Delta}{2})}\right)$$

③ $t > +\frac{\Delta}{2}$

$$y(t) = \int_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} A\alpha e^{-\alpha(t-\tau)} d\tau =$$

$$= A\alpha e^{-\alpha\frac{\Delta}{2}}\left(\frac{1}{\alpha} e^{\alpha\tau}\Big|_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}}\right) =$$

$$= Ae^{\alpha\frac{\Delta}{2}} e^{-\alpha t}\left(1-e^{-\alpha\Delta}\right) = Ae^{-\alpha(t-\frac{\Delta}{2})}\left(1-e^{-\alpha\Delta}\right)$$



Лекция 30.10

Балансная модуляция

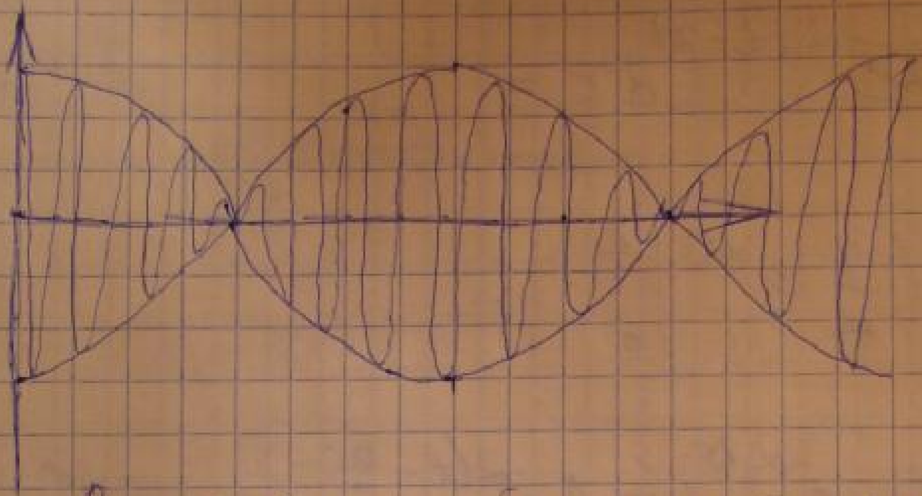
Бал. модуляция — м. путем перемножения сигналов

$$y(t) = s(t)\,x(t)$$

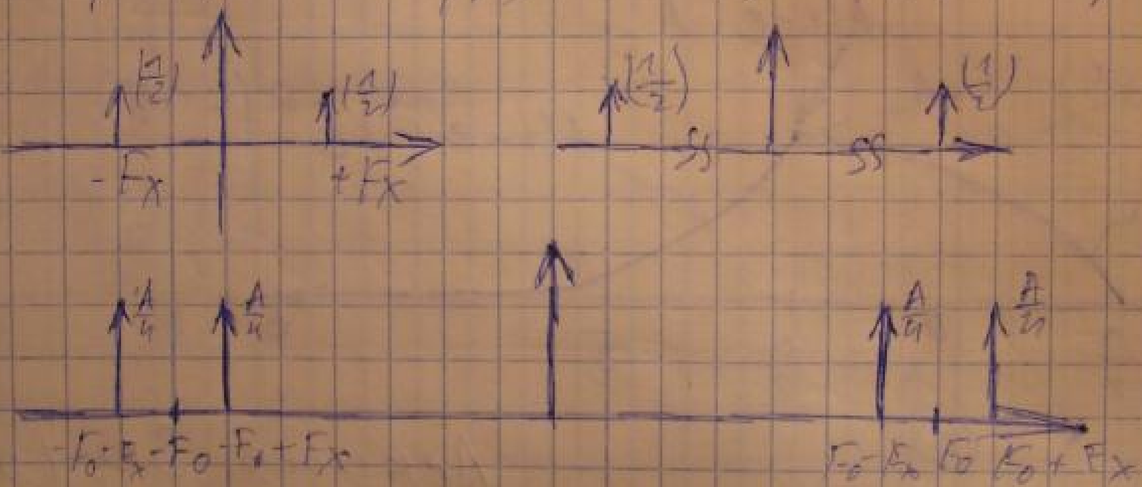
$$s(t) = A\cos(2\pi F_0 t)$$

$$x(t) = A\cos(2\pi f_x t)$$

$y(t) = A\cos(2\pi f_x t)\, A\cos(2\pi F_0 t)$



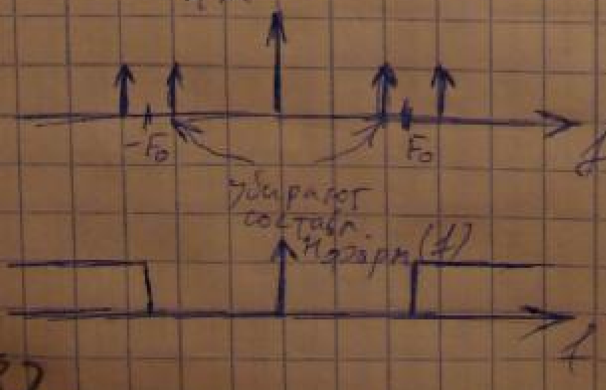
В случае балансной модуляции требуется другой демодулятор



$\eta_{max\ КАМ} = 1/3$

Однополосная АМ

$\Delta F_{\text{сх пол. АМ}} = 2F_x$



Удаётся сэкономить полосу, но схема демодулятора будет ещё сложнее.

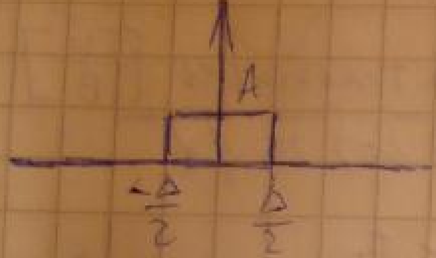
Модуляция гарм. сигнала произв. сигналом

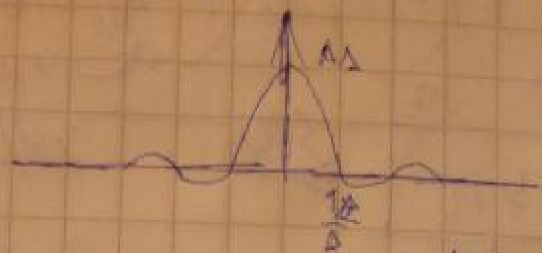
Радиоимпульсы

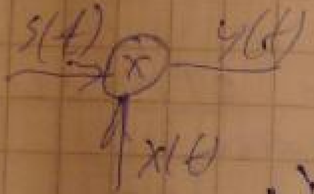
Сигналы

Видео — $S(f) \to 0$

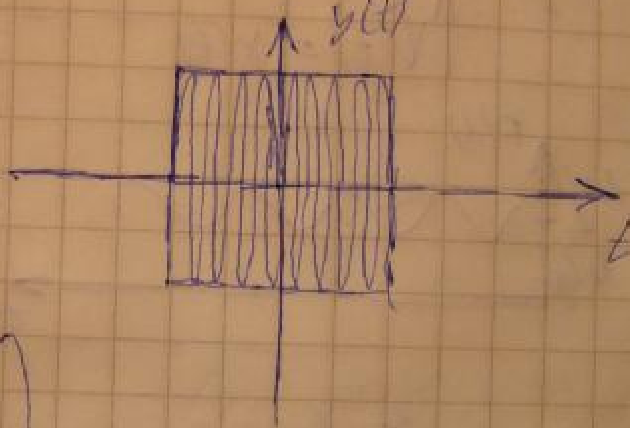
Радио — $S(f) \to F_0$







$y(t) \neq$



Угловая модуляция

$s(t) = A\cos(\Phi(t))$

← угол (полная фаза)

1) $\varphi(t) = const \Rightarrow s(t) = const$

2) $\varphi(t) = 2\pi f t + \varphi_0$ $\quad s(t) = A\cos(2\pi f t + \varphi)$

$F = \frac{1}{2\pi} \frac{\varphi(t) + \varphi_0}{t} \Leftarrow \varphi(t) = 2\pi F t + \varphi_0$

$\varphi(t)$ – люб.)

$$f = \frac{1}{2\pi}\frac{d\Phi(t)}{dt}$$

$$\Phi(t) = \int_0^t 2\pi F(\tau)\,d\tau + \varphi_0$$

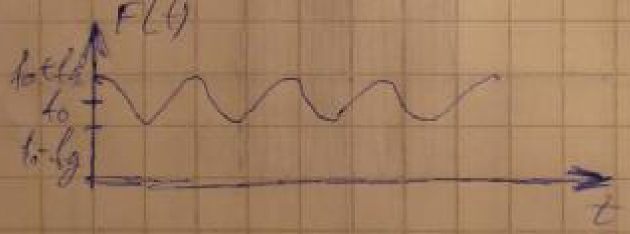
Частотная модуляция

$$s_{чм}(t) \Rightarrow F(t) = f_0 + k_{чм}\, x(t)$$

$k_{чм}$ – чувствительность $\left[\frac{Гц}{В}\right]$

$$x(t) = A\cos 2\pi f t$$

$$f \ll f_0$$



Макс. отклон. частоты сигнала – девиация частоты

$$\Phi(t) = \int_0^t 2\pi\, k_{чм}\cdot A(\tau)\,d\tau + \cancel{\varphi_0}^{\,0} =$$

$$= \int_0^t 2\pi\left(f_0 + k_{чм}A\cos 2\pi F t\right)d\tau =$$

$$= \int_0^t 2\pi f_0\,d\tau + \int_0^t 2\pi k_{чм}\cos(2\pi F\tau)\,d\tau =$$

$$= 2\pi f_0 t + \frac{2\pi k_{чм}A}{2\pi F}\sin(2\pi F t)\Big|_0^t =$$

$$= 2\pi f_0 t + \frac{k_{чм}A}{F}\sin(2\pi F t)$$

$$S(t) = C\cos\left(2\pi f_0 t + \frac{k_{чм}A}{F}\sin(2\pi F t)\right)$$

$$F(t) \to \Phi(t) \to S(t)$$

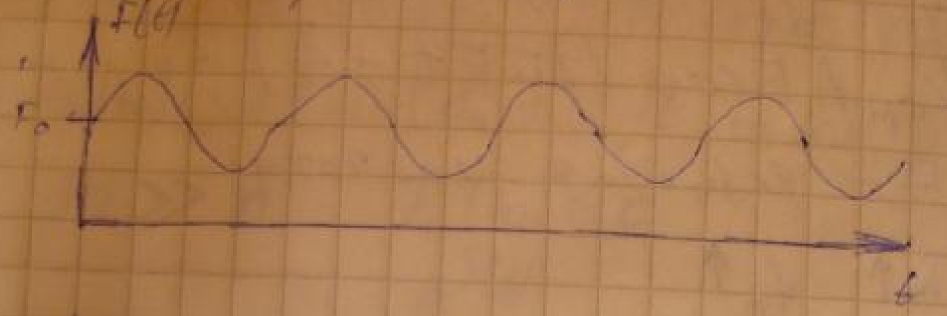
84 $k_{чм}A$ – девиация ч.

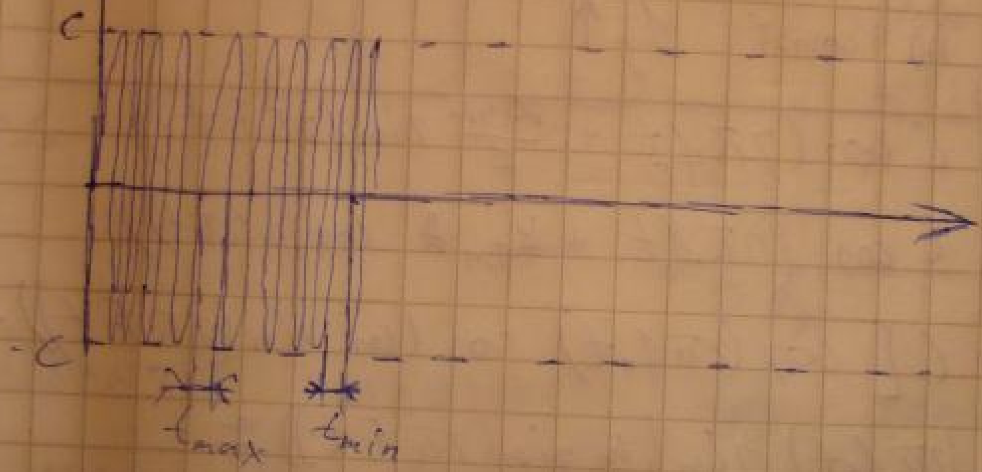
$\frac{F_g}{F} = m$ — индекс угловой мод.

ЧМ: $(A, F) \to (F_g, m)$

$A\uparrow \Rightarrow F_g\uparrow, m\uparrow$

$F\uparrow \Rightarrow F_g = const, m\downarrow$





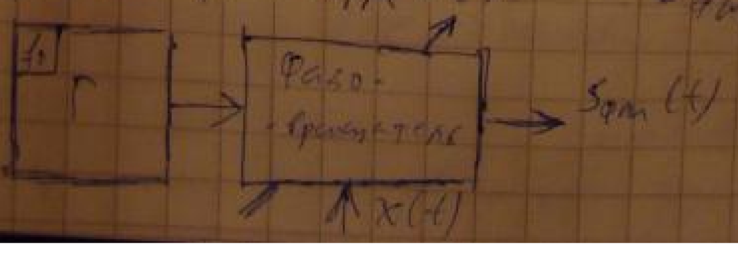
$$t_{min} = \frac{1}{F_0 - F_g}$$

$$t_{max} = \frac{1}{F_0 + F_g} \qquad f_0 \mp f_g = \frac{1}{T_{min}}$$

ФМ

$$\varphi(t) = 2\pi f_0 t + \varphi_0(t)$$

$$\varphi_0(t) = \varphi_0 + k_{ФМ}\, x(t) \qquad k_{ФМ} = \left[\frac{рад}{В}\right]$$



$\Phi(t) = 2\pi f_0 t + \varphi_0 + K_{\Phi M} A \cos(2\pi f t)$

$s(t) = C \cos(2\pi F_0 t + K_{\Phi M} A \cos(2\pi f t))$

$K_{\Phi M} A = m$ – индекс угловой модуляции

$F(t) = \frac{1}{2\pi} \frac{d\Phi(t)}{dt} = f_0 + K_{\Phi M} A F(-\sin(2\pi F t)) =$

$= f_0 + K_{\Phi M} A F \cos(2\pi F t + \frac{\pi}{2})$

$A K_{\Phi M} F$ – девиация частоты

$A \uparrow \to m \uparrow \quad f_g \uparrow$

$F \uparrow \to m = const \quad f_g \uparrow$

ЧМ: $s(t) = C \cos(2\pi f_0 t + \frac{K_{ЧМ}}{F} \sin(2\pi F t))$

ФМ: $s(t) = C \cos(2\pi f_0 t + K_{\Phi M} A \cos(2\pi F t))$

$s(t)_{ЧМ} = C[\cos(2\pi f_0 t) \cos(m \sin(2\pi F t)) -$

$- \sin(2\pi f_0 t) \sin(m \sin(2\pi F t)) =$

$\cos(m \cos(2\pi F t)) = J_0(m) + 2 J_2(m) \cos(2\pi 2 F t) +$

$+ J_4(m) \cos(2\pi 4 F t)$

$J(m)$ – функ. Бесселя

$\sin(m \sin(2\pi F t)) =$

$= 2J_1(m)\cos(2\pi F t) + 2J_3(m)\cos(2\pi 3F t)$

$S_{чм}(t) = C[J_0(m)\cos(2\pi F t) +$

$+ J_1(m)[\cos(2\pi(f_0+F)t] - \cos(2\pi(f_0-F)t] +$

$+ J_2(m)[\cos 2\pi(f_0+2F)t + \cos(2\pi(f_0+2F)t] +$

$+ J_3(m)[\cos 2\pi(f_0+3F)t - \cos(2\pi(f_0-3F)t]$

$m \gg n \quad J_n(m) \approx 0$

Характер спектров при тонально-
-мод. ЧМ и ФМ сигналах (с учет. гармоники)

1) Сигнал раскл. в РФ ⇒ его спектр сост. из $\delta(f)$ (т.к. сигн. периодич.)

2) δ-фу-й бескон. много [illegible] на част. $f_0 \pm Kf$

3) Веса δ-фу-й опр. с помощью фу-и Бесселя

4) Индекс угл. модуляции m опр. до какого номера гармоники будут значимыми

Полоса частот

$\Delta f \propto 2m f$ т.э.о

Преимущества Угловой мод.

1) Большая помехоустойчивость

2) Выс. КПД (теорет. - 100%)

Комплексная огибающая

лекция б.11

$e^{jx} = \cos x - j\sin x$

$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

Св-ва симметрии:

$x(t) = Re(x(t))$

$X(f)$ - симметр

$Re\,X(f) = Ev(f)$

$Im\,X(f) = Odd(f)$

$x(t)$ - комплекс.

! $\dot{x}(t)$

$X(f)$ / $\dot{X}(f)$ - спектр компл. сигнала (не облад. симметрией)

$\dot{x}(t) \rightarrow$ сист $\rightarrow \dot{s}(t)$

88

$\dot{x}(t) = \mathrm{Re}\,\dot{x}(t) + j\,\mathrm{Im}\,\dot{x}(t)$

$\dot{y}(t) = \mathrm{Re}\,\dot{y}(t) + j\,\mathrm{Im}\,\dot{y}(t)$

$\cancel{\dot{x}(t) = \alpha \dot{y}}$ $\dot{y}(t) = \alpha\,\dot{x}(t)$

$\dot{y}(t) = \alpha\,\mathrm{Re}\,\dot{x}(t) + j\,\alpha\,\mathrm{Im}\,\dot{x}(t)$

$\dot{z} = \dot{x} + \dot{y}$

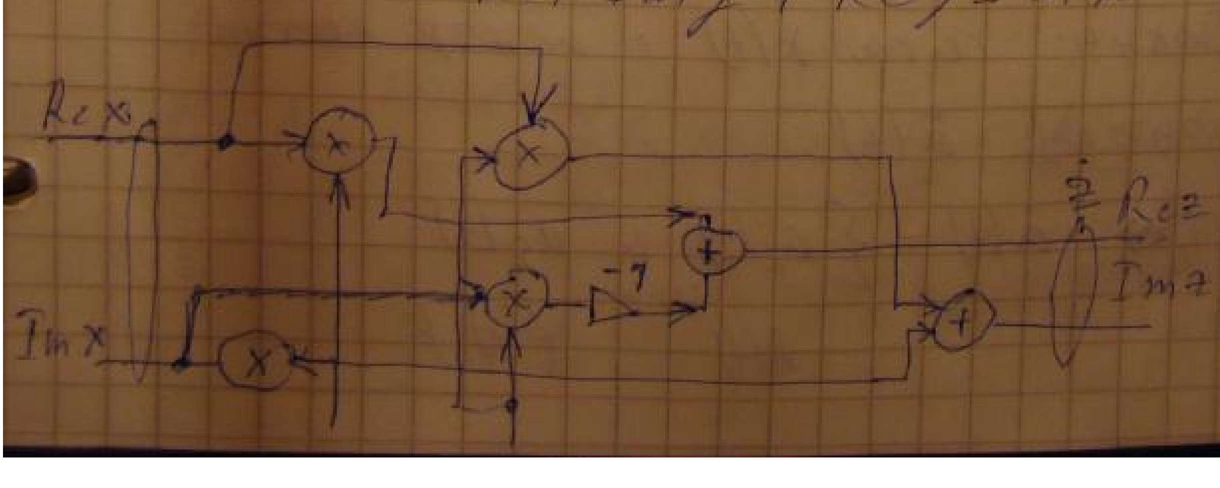
$\dot{z} = \mathrm{Re}\,\dot{x} + \mathrm{Re}\,\dot{y} + j(\mathrm{Im}\,\dot{x} + \mathrm{Im}\,\dot{y})$

$\dot{z} = \dot{x} \cdot \dot{y}$

$\dot{z} = (\mathrm{Re}\,x + j\,\mathrm{Im}\,x)(\mathrm{Re}\,y + j\,\mathrm{Im}\,y) =$

$\mathrm{Re}\,\dot{z} = \mathrm{Re}\,x\,\mathrm{Re}\,y - \mathrm{Im}\,x\,\mathrm{Im}\,y$

$\mathrm{Im}\,z = \mathrm{Re}\,x\,\mathrm{Im}\,y + \mathrm{Re}\,y\,\mathrm{Im}\,x$



Сигналы со сложной модуляцией

$x(t) = A(t)\cos(\omega_0 t + \varphi(t))$

$\omega_0 = 2\pi f_0$

2 вопр. – как сформировать такой сигнал

Как по такому сигналу определить эти две составляющие $A(t), \varphi(t)$

$$x(t) = \underbrace{A(t)\cos(\varphi(t))}_{a_c(t)}\cos(\omega_0 t) - \underbrace{A(t)\sin(\varphi(t))}_{a_s(t)}\sin\omega_0 t$$

$a_c(t) = A(t)\cos\varphi(t)$

$a_s(t) = -A(t)\sin\varphi(t)$

$x(t) = a_c(t)\cos(\omega_0 t) + a_s(t)\sin\omega_0 t$

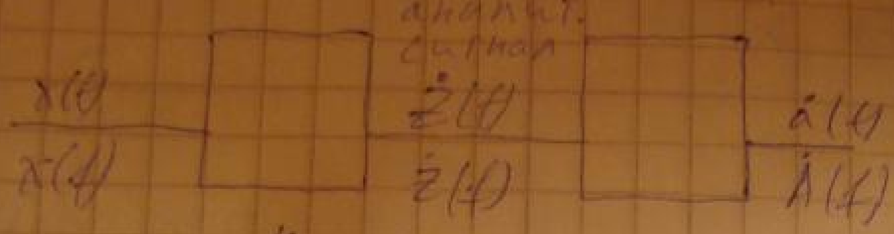
$I = a_c(t)$ – in phase
синфазный

$Q = a_s(t)$ – quadrature
квадратурный

Получ. сигн. $x(t)$ с бескр. компл. функц. $\mathring{a}(t)$

$x(t) \leftrightarrow \mathring{a}(t) = \{a_c(t)$ и $a_s(t)$
или
$A(t)$ $\varphi(t)\}$

$x(t) \to \dot{a}(t)$



$x(t) = A_0 \cos(\omega_0 t + \varphi_0)$

$A(t) = const$

$\varphi(t) = const$

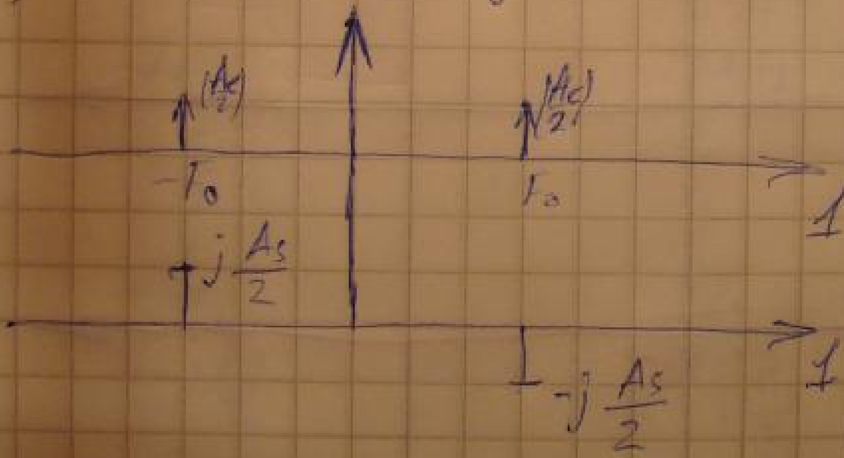
$x = A_0 \cos(\omega_0 t + \varphi_0) = A_0 \cos\varphi_0 \cos\omega_0 t +$

$+ A_0 \sin\varphi_0 \sin\omega_0 t =$

$= A_c \cos\omega_0 t + A_s \sin\omega_0 t$

$\dot{X}(f) = \mathrm{Re}\, X(f) + j\, \mathrm{Im}\, X(f)$

$A_s = -A_0 \sin\varphi_0$



Аналит. сигн. – действ. часть ком-
плекса зап.

$\dot{z}(t) = A_0 e^{j(\omega_0 t + \varphi_0)} = \underbrace{A_0 \cos(\omega_0 t + \varphi_0)}_{\mathrm{Re}\,\dot{z} = x} + j \underbrace{A_0 \sin(\omega_0 t + \varphi_0)}_{\hat{x} \text{ – сопряж. сигнал}}$

Преобр. Гильберта

$$\text{ПГ от } x(t) = \hat{x}$$

$$j\hat{x} = z - x$$

$$\hat{x} = j(x - z)$$

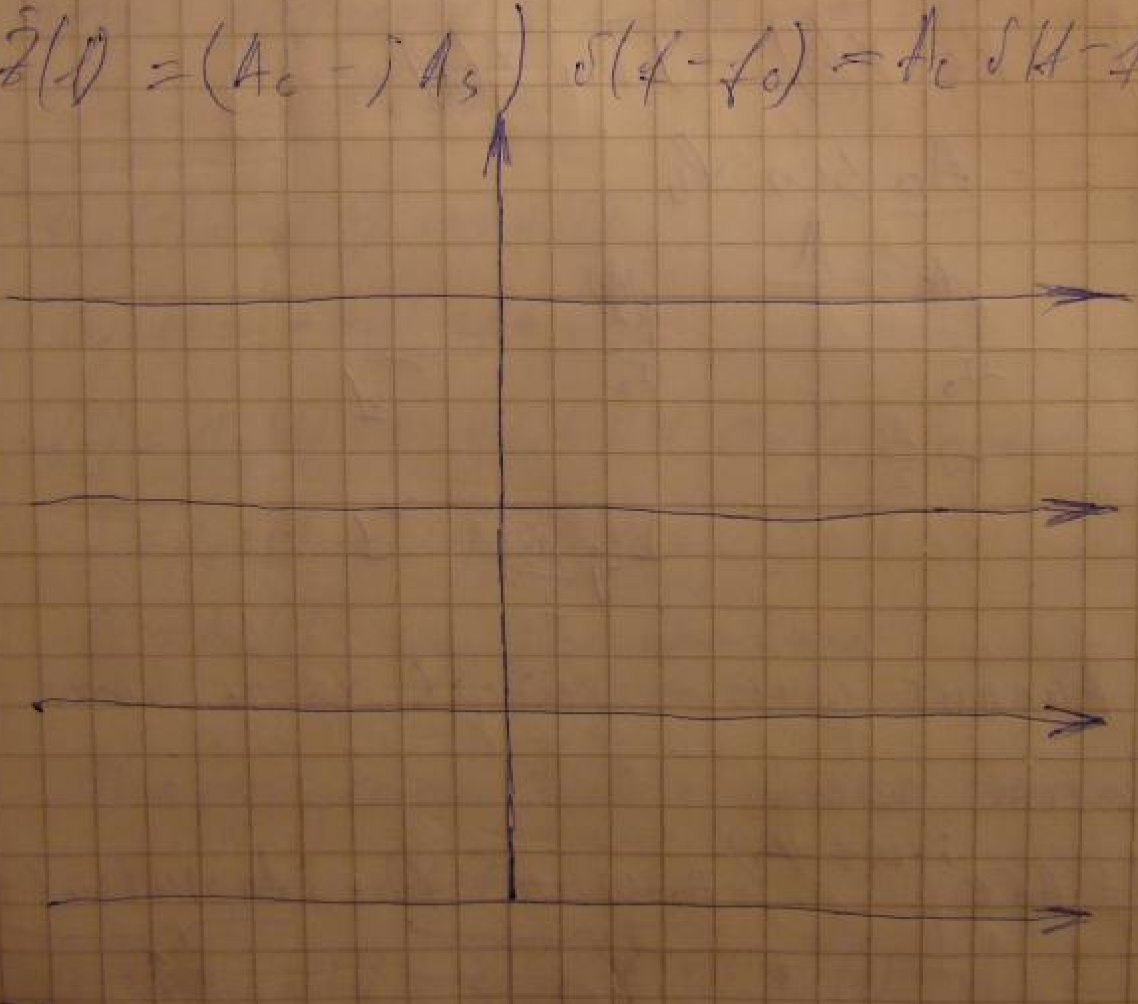
$$z(t) = x(t) + j\hat{x}(t)$$

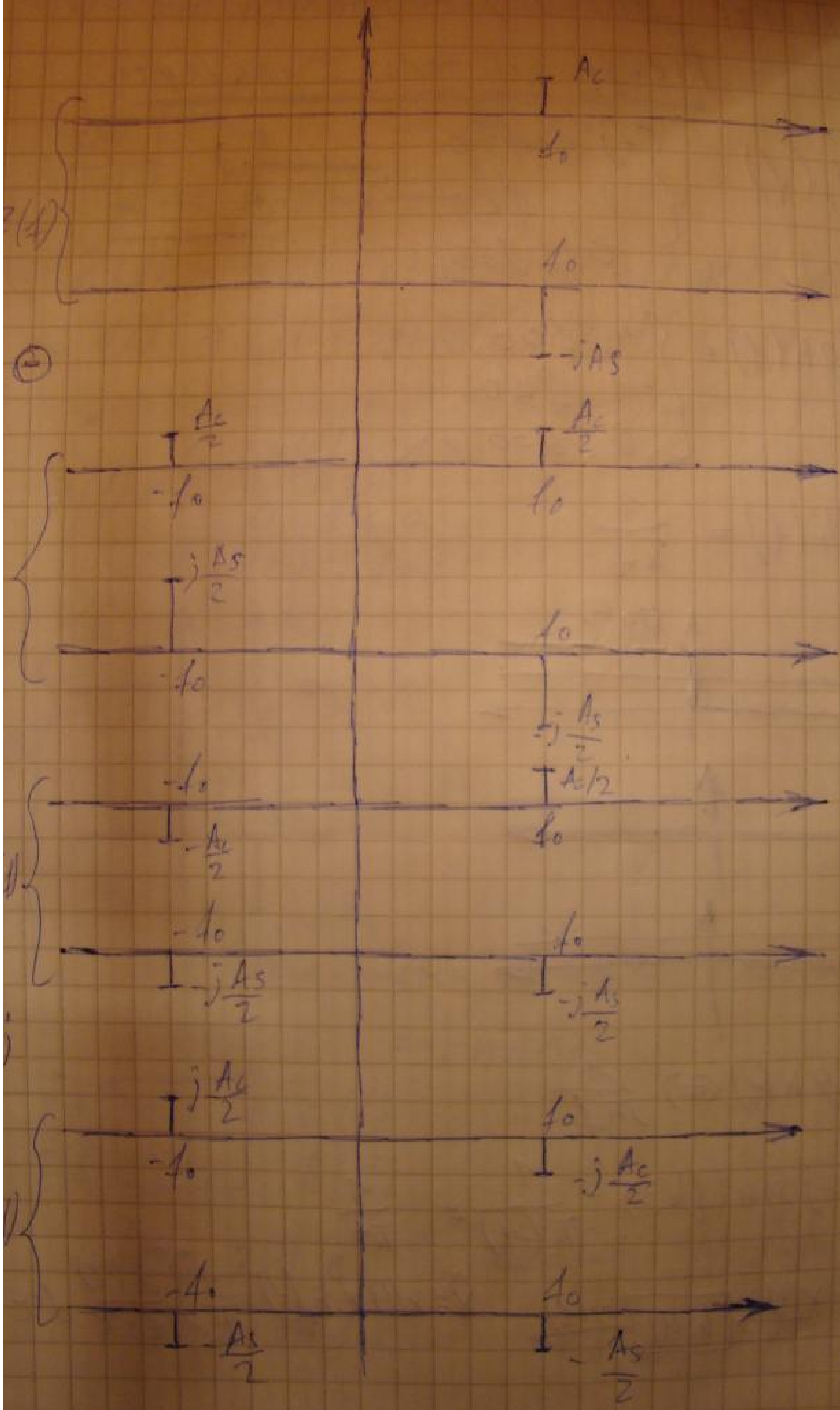
$$j\hat{x}(t) = z(t) - x(t)$$

$$x(t) = A_0 e^{j(\omega_0 t + \varphi_0)} = A_0 e^{j\varphi_0} e^{j\omega_0 t} =$$

$$= (\underbrace{A_0 \cos\varphi_0}_{A_c} + j\,\underbrace{A_0 \sin\varphi_0}_{A_s})\, e^{j\omega_0 t}$$

$$\hat{z}(f) = (A_c - jA_s)\,\delta(f - f_0) = A_c\,\delta(f - f_0) - jA_s\,\delta(f$$





Ac
f0
f0
-jAs
Ac/2
Ac/2
-f0
f0
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-j As/2
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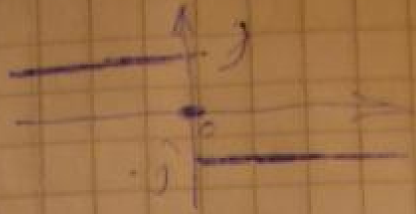
-j Ac/2
-f0
f0
-As/2
-As/2

Спектр антисимметричн

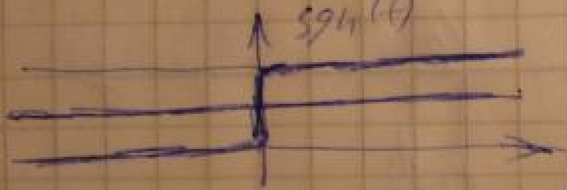
Преобразователь Гильберта

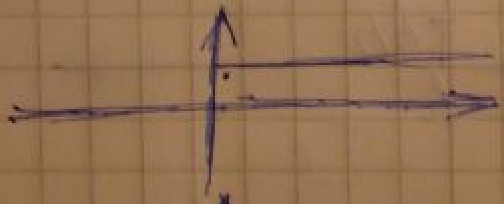
$$H_r(f) = \begin{cases} -j, & f>0 \\ j, & f<0 \\ 0, & f=0 \end{cases}$$



$$\operatorname{sgn}(f) = \begin{cases} 1, & f>0 \\ -1, & f<0 \\ 0, & f=0 \end{cases}$$

$$h_r(t) = \frac{1}{\pi t}$$



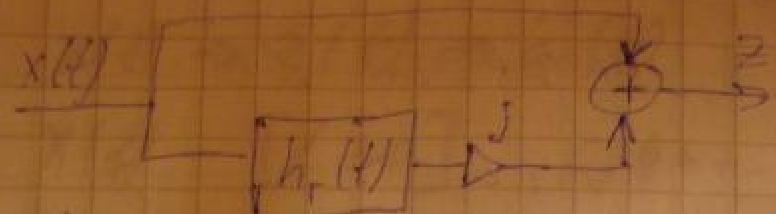


$$\operatorname{sgn}(t) \Leftrightarrow \frac{1}{j\pi f}$$

$$-j\operatorname{sgn}(f) \Leftrightarrow -\frac{1}{\pi(-t)} = \frac{1}{\pi t}$$

$$x(t) \rightarrow \boxed{h_r(t)} \rightarrow y(t) \qquad \hat{x} = x(t) * h_r(t) = \frac{1}{\pi}\int_{-\infty}^{+\infty} \frac{x(\tau)}{t-\tau}\, d\tau$$

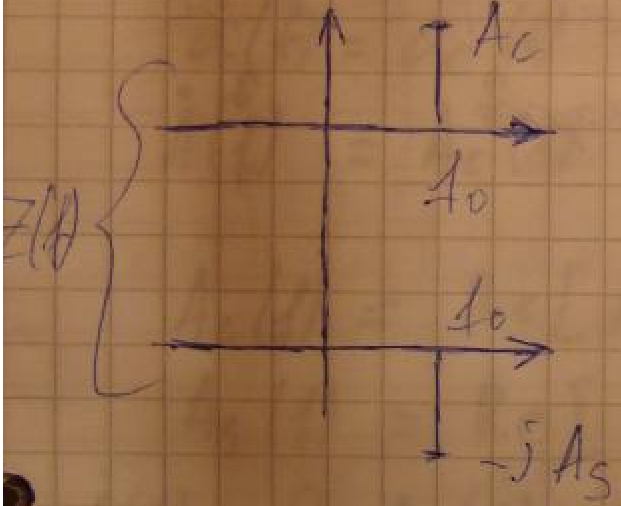
$z(t) = x(t) + j\hat{x}(t)$



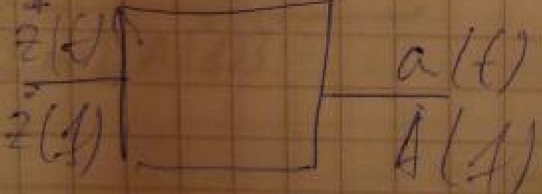
$\dot{Z}(f) = X(f) + j\hat{X}(f) = X(f) + jX(f)H_1(f) =$

$= X(f)[1 + j(-j \cdot sgn(f))] = X(f)[1 + sgn(f)] =$

$$Z(f) = \begin{cases} 2X(f) & f > 0 \\ 0 & f < 0 \\ X(0), & f = 0 \end{cases}$$



Извлачење КО из аналит. сигнала



$\dot{a}(t) = \dot{z}(t)\, e^{-j\omega_0 t}$

$\dot{A}(f) = Z(f + f_0)$

$$A(f) = \begin{cases} 2A(f+f_0) & f \geq -f_0 \\ 0 & f < -f_0 \\ \dot{A}(-f_0) & f = -f_0 \end{cases}$$

$z(t) = (A_c - jA_s)\, e^{j\omega_0 t}$

$\dot{a}(t) = A_c - jA_s$

Для получ. спектра КО

надо сдел. 2 шага

1) обнулить левую пол. спектра
удвоить правую

2) Перенести спектр влево

на нулевую частоту.

~~Это~~ с Взятие КО – это

метод превращение РС в ВС

без потери информ

Лекция 13.11

422б 15:00 2 этаж

$x(t) \to z(t) \to \dot{a}(t)$

$x(f) - [f \geq 0]$ сдвин. на f_0 влево

$$x(t) = \underset{\text{const}}{A} \cos(\omega_0 t + \underset{\text{const}}{\varphi})$$

$$x(t) = a_c \cos \omega t + a_s \sin \omega t$$

$$a_c = A \cos \varphi$$

$$a_s = -A \sin \varphi$$

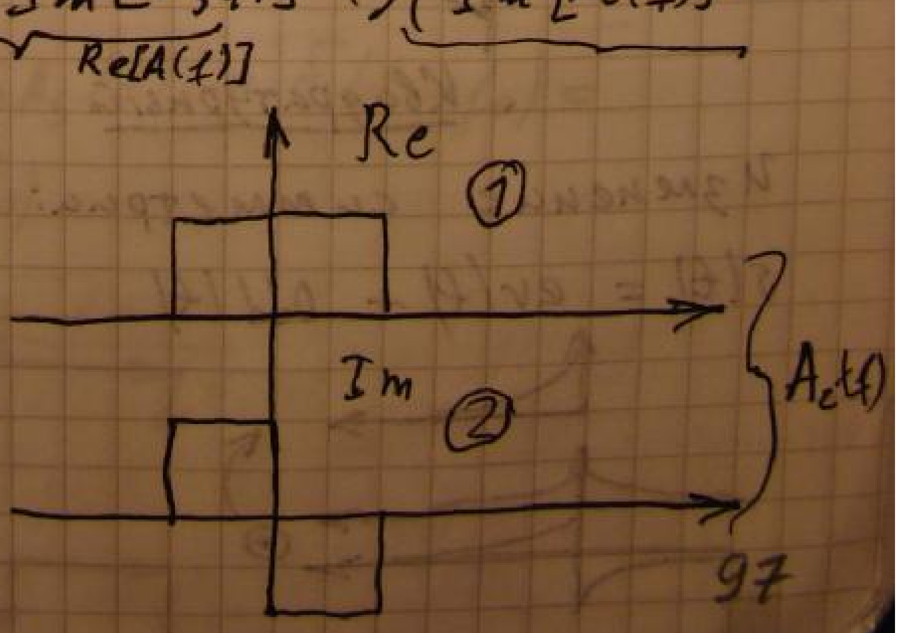
$$\dot{a}(t) = a_c(t) - j\, a_s(t)$$

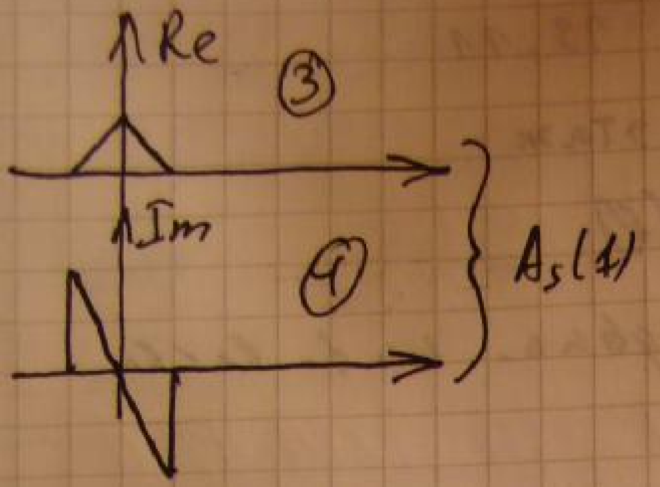
$$\dot{A}(f) = A_c(f) - j\, A_s(f)$$ – спектр компл. огибающей

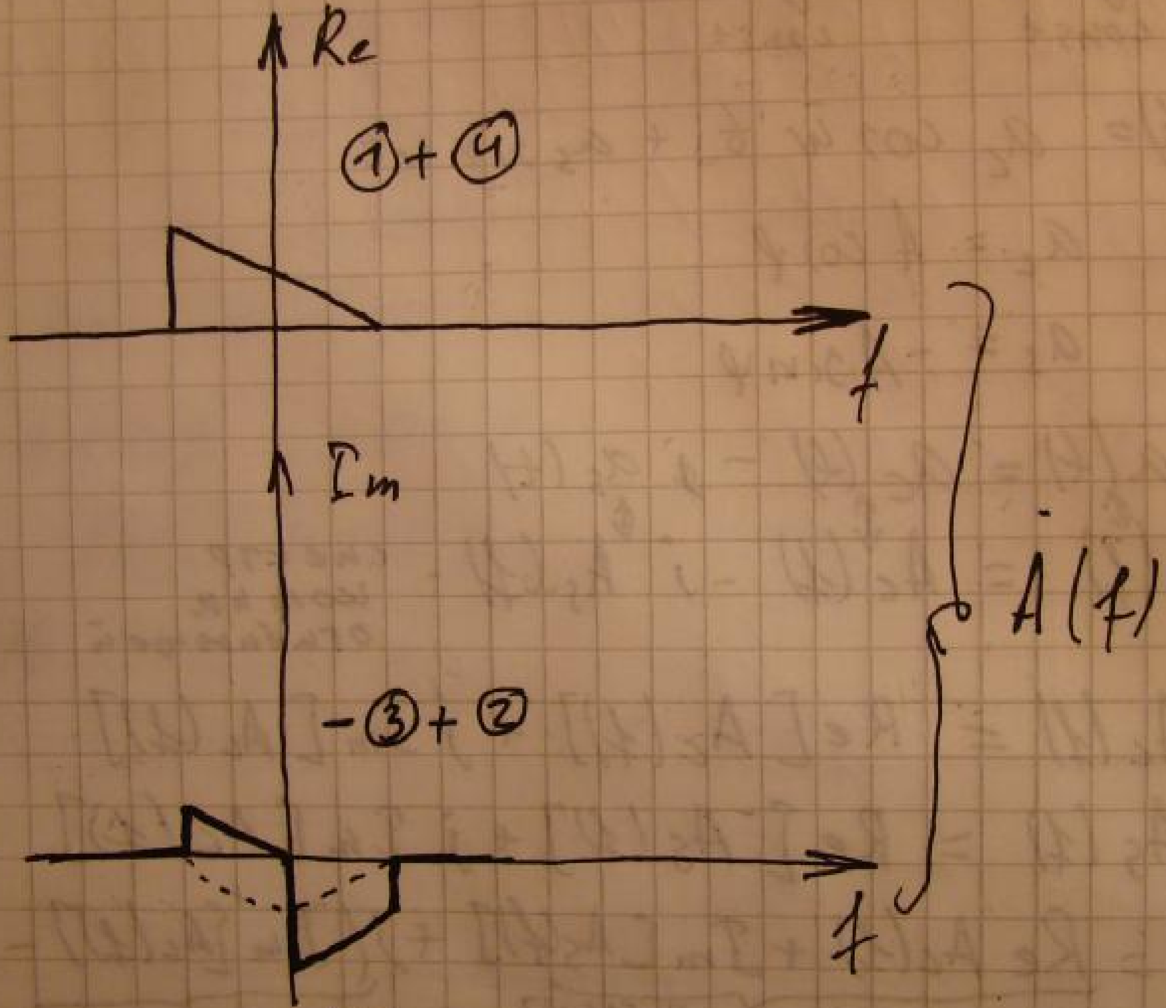
$$A_c(f) = Re[A_c(f)] + j\, Im[A_c(f)]$$

$$A_s(f) = Re[A_s(f)] + j\, Im[A_s(f)]$$

$$\dot{A}(f) = \underbrace{Re\, A_c(f) + Im[A_s(f)]}_{Re[A(f)]} + j\big(\underbrace{Im[A_c(f)] - Re[A_s(f)]}_{Im[A(f)]}\big)$$



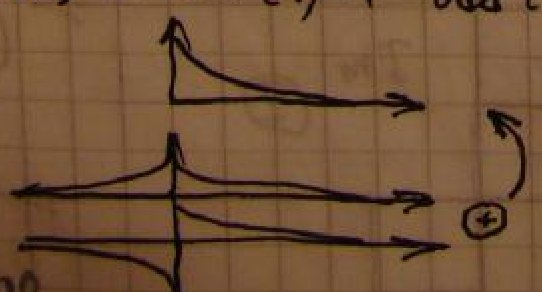




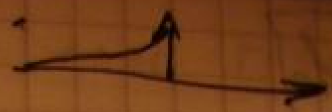
<u>Квадратурный</u> <u>модулятор</u>

Изменение симметрии:

$s(t) = ev(t) + odd(t)$

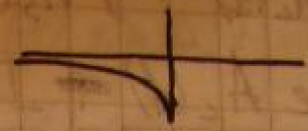


$s(t) = ev(t) - odd(t)$

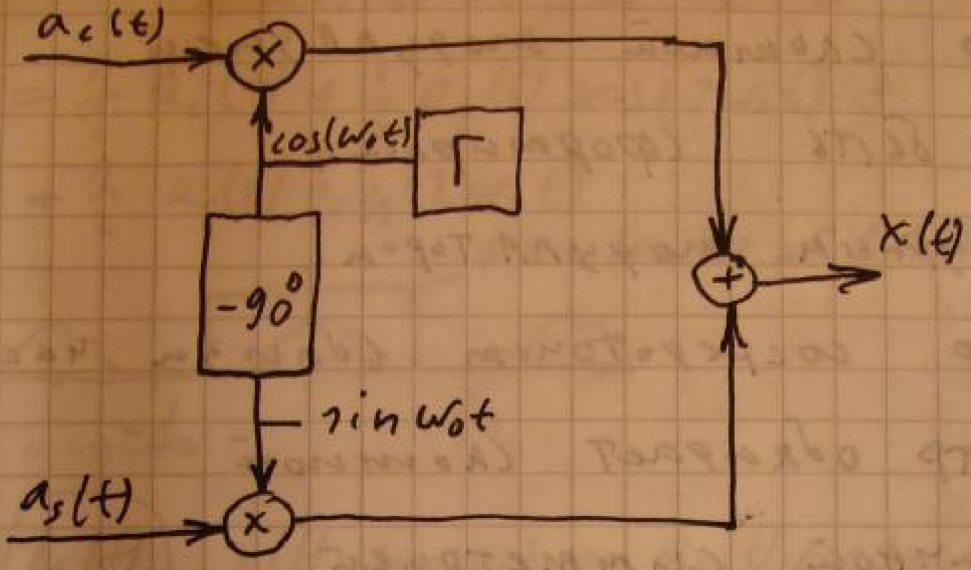


$s_1(t) = s(-t)$

$s_2(t) = -ev(t) + odd(t)$



$s_2(t) = -s(-t)$



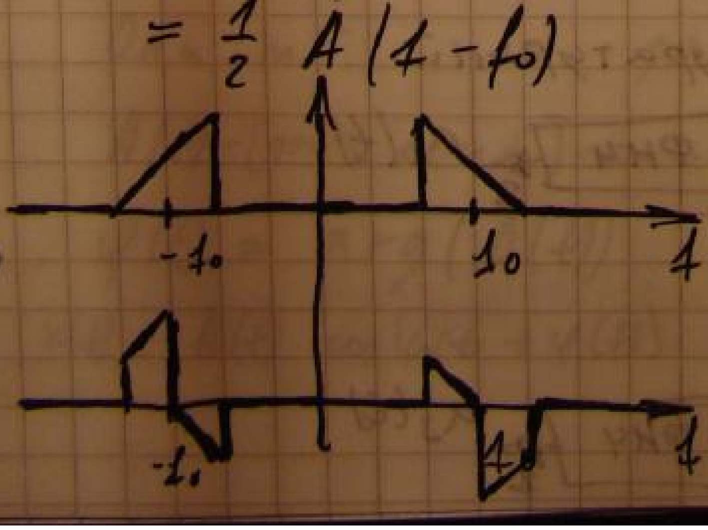
$x(t) = a_c \cos \omega t + a_s \sin(\omega t)$

$x(f) = \frac{1}{2} A_c(f) * \underbrace{\delta(f \mp f_0)}_{\text{спектр cos}} + \frac{1}{2} j A_s(f) * \underbrace{\mp\delta(f \mp f_0)}_{\text{спектр sin}}$

1) $f \approx f_0$ $\quad \frac{1}{2} A_c(f) * \delta(f - f_0) + j \frac{1}{2} A_s(f) * -\delta(f - f_0) =$

$= \frac{1}{2} \left(A_c(f) - j A_s(f) \right) * \delta(f - f_0) =$

$= \frac{1}{2} \dot{A}(f - f_0)$



2) $\boxed{f \simeq f_0}$

$$\frac{1}{2}\left(A_c(f) + j A_s(f)\right) * \delta(f + f_0) =$$

$$= \frac{1}{2}\left\{ \underbrace{\mathrm{Re}\, A_c(f) - \mathrm{Im}\, A_s(f)}_{\text{разворот } \mathrm{Re}\, \dot{A}} + j\,[A_c(f) + \mathrm{Im}\, A_s(f))\right\} *$$

$$* \ \delta(f + f_0)$$

РС со сложной модуляцией может быть сформирован квадратурным модулятором

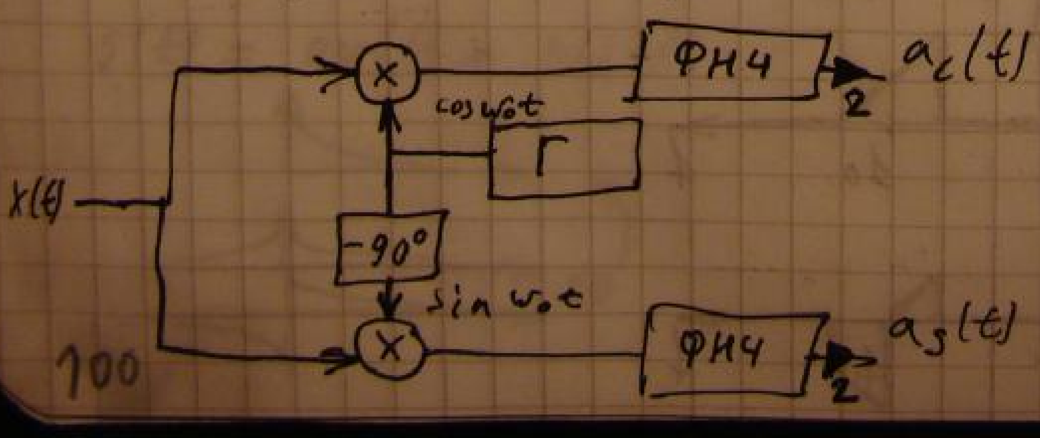
Спектр сосредоточен вблизи частот f_0

Спектр обладает сложной иерархичной симметрией

Вблизи частоты f_0 спектр повт. по форме спектр КО и меньше его в 2 раза

Извлечение КО из радио сигнала

Демодулятор квадратурный



Будем считать что мы точно знаем несущую частоту f_0

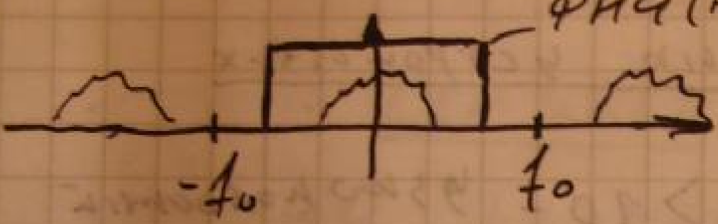
$x(t) = a_c(t) \cos w_0 t + a_s(t) \sin w_0 t$

$A_c(t)$ $\quad x(t) \cdot \cos w_0 t = a_c(t) \cos^2 w_0 t + a_s(t) \times$

$\times \sin w_0 t \cos w_0 t =$

$= \frac{1}{2} \dot{a}_c(t) + \frac{1}{2} a_c(t) \cos(2 w_0 t) +$

$+ \frac{1}{2} a_c(t) \sin(2 w_0 t)$



$A_s(t)$ $\quad x(t) \cdot \sin w_0 t = a_c(t) \cos(w_0 t) \sin(w_0 t) +$

$+ a_s(t) \sin^2(w_0 t) = \frac{1}{2} a_c(t) \sin(2 w_0 t) +$

$+ \frac{1}{2} a_s(t) - \frac{1}{2} a_s(t) \cos(2 w_0 t)$

Если исх сигнал это РС то

с помощью кв. демодулятора

могут быть получ. кв. состав.

Они м.б. исп. для форм. КО

$A(t)$ - [огиб. огиб.] $= |\dot{A}(t)| = \sqrt{a_c^2 + a_s^2}$

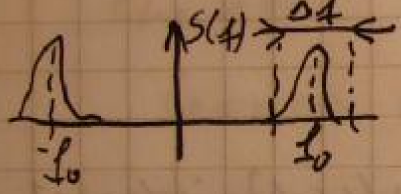
$\varphi(t) = \arg(\dot{A}(t))$

$x(t) = A(t) \cos(w t + \varphi(t))$

Применение КО

1) Формирова РС со сложн. модуляцией
2) Демодуляция сигнала с АМ, УМ, сложн. мод.
3) Анализ преобраз Радиосигналов

Преобразования РС в линейных устройствах



$\frac{f_0}{\Delta f} > 10$ - узкополосный РС

$\frac{f_0}{\Delta f} < 3$ широкополосный РС

$S(t)$ - УРС

УЛС - узкополосная лин. система.

Раб. полоса ЛС долж быть согласована с полосой спектра сигнала.

Тогда возм прим. анализа с примен. КО

$x(t) \to \boxed{УЛС} \to y(t)$

$\downarrow$ ko(x(t))

$\dot{A}_x(t) \xrightarrow{h(t),\, h(t),\, H(p)} \boxed{ЭС} \to \dot{A}_y(t)$

AM

$f = 75$ кГц

$F_0 = 150$ МГц

$T = 1$ с

$N = 75 \cdot 10^3$ периодов

$N_{точ.} = 750 \cdot 10^3$ точек

$N' \approx 7.5$ млрд

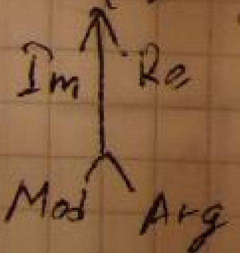
Семинар 15.11

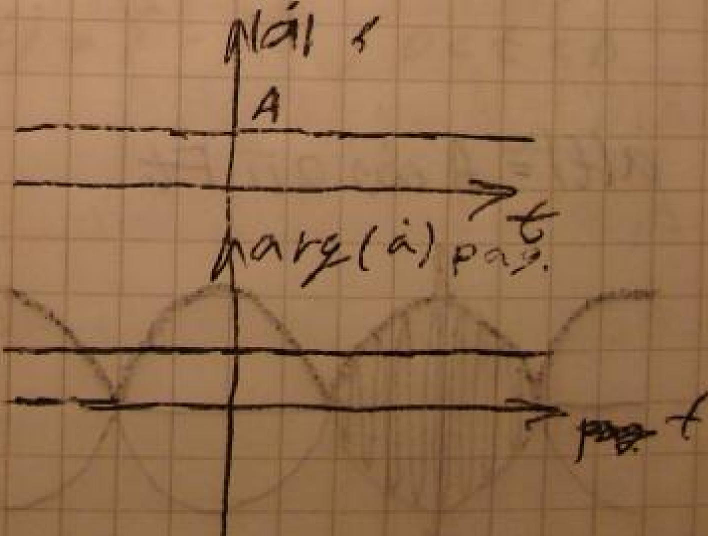
КО примера нахождения

$x(t) = A\cos(\omega t)$

$\dot{X}(t) = Ae^{j\varphi}$

$\dot{a}(t) = \underbrace{A}_{Mod}\underbrace{e^{j\varphi}}_{Arg}$

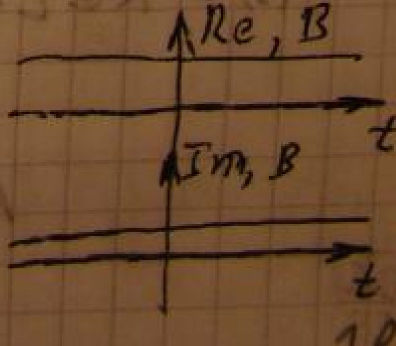




$\Delta F = \frac{1}{2\pi}\frac{d[\arg(\dot{a}(t))]}{dt}$

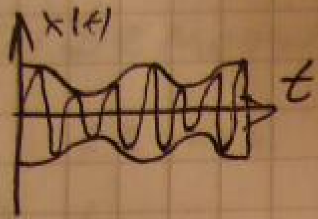
$\Delta f(t) = 0$

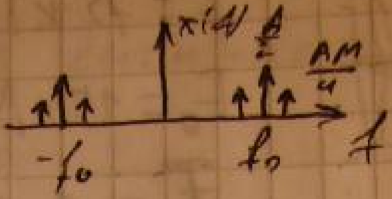
$\dot{a}(t) = \underbrace{A\cos\varphi}_{Re} + j\underbrace{A\sin\varphi}_{Im}$

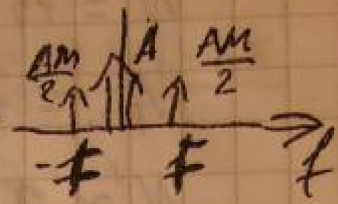


$x(t) = A\cos\varphi\cos(\omega_0 t) - A\sin\varphi\sin(\omega_0 t)$

2. $x(t) = A(1 + M\cos 2\pi F t)\cos 2\pi f_0 t$ AM

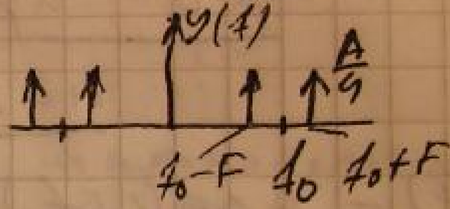


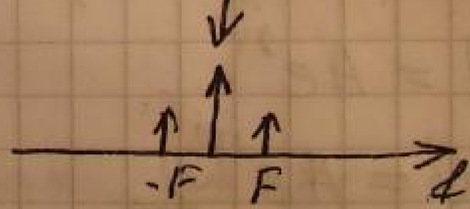




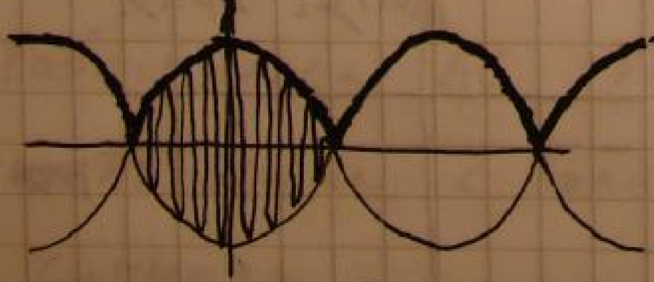
$\dot{a}(t) = A(1 + M\cos 2\pi F t)$

3. $y(t) = A\cos 2\pi F t \cos 2\pi f_0 t$ Ban AM





$\dot{a}(t) = A\cos 2\pi F t$



$$y(t) = \mathrm{Re}\left[A\cos 2\pi F t \cdot e^{j2\pi f_0 t}\right]$$

$$\dot{z}(t)$$

$$\times e^{-j2\pi f_0 t}$$

$$A\cos 2\pi F t = \dot{a}_y(t)$$

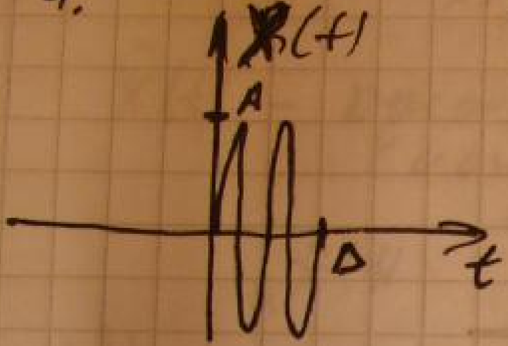
$$s(t) = A \sin 2\pi F t \sin 2\pi f_0 t$$

$$s(t) = A \sin 2\pi F t \cdot \cos\left(2\pi f_0 t - \frac{\pi}{2}\right) =$$

$$= Re\left[A \sin 2\pi F t \cdot e^{j\left(2\pi f_0 t - \frac{\pi}{2}\right)}\right]$$

$$a_s(t) = -jA \sin 2\pi F t$$

4.

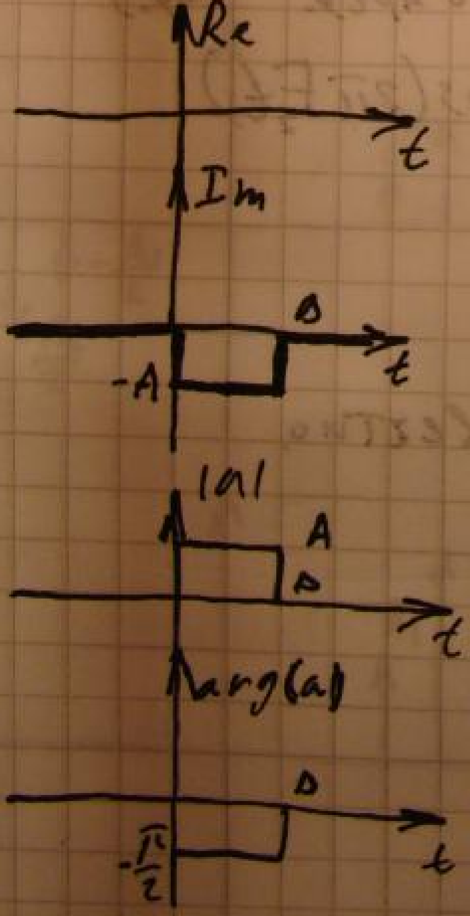


$$x(t) = \begin{cases} A \sin 2\pi \frac{3}{\Delta} t, & 0<t<\Delta \\ 0, & \text{ост. } t \end{cases}$$

$$F_0 = \frac{3}{\Delta}$$

$0<t<\Delta$ $$x(t) = Re\left[A e^{j\left(2\pi \frac{3}{\Delta} t - \frac{\pi}{2}\right)}\right]$$

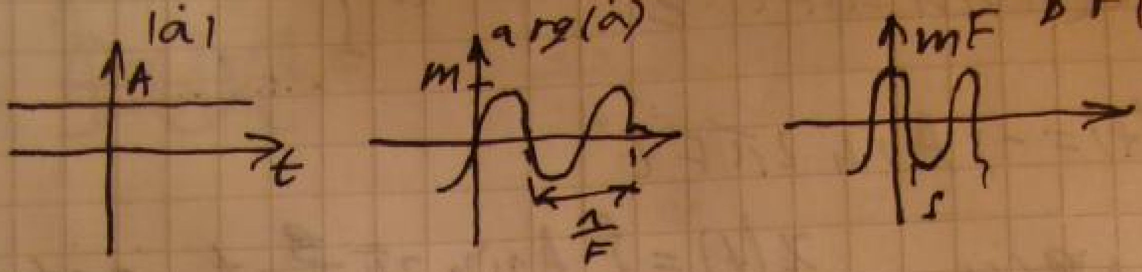
$$\dot{a}(t) = -jA$$

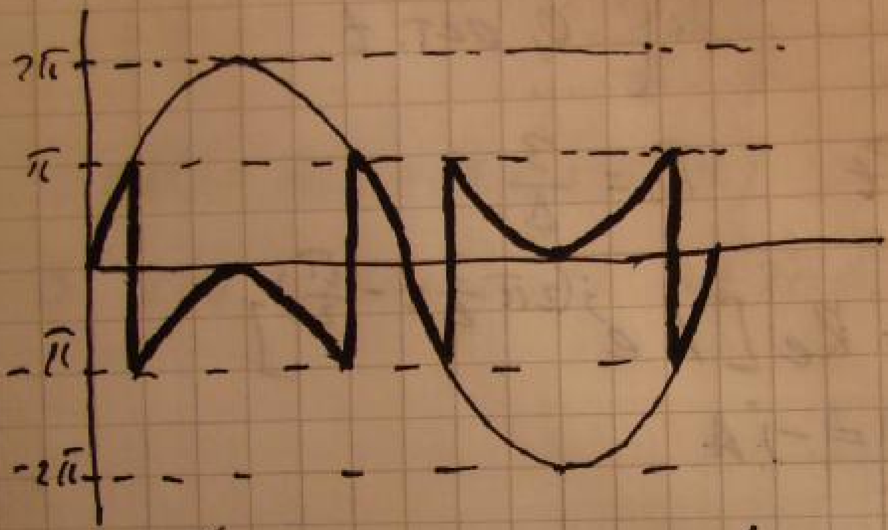


$$\dot{a}(t) = \begin{cases} -jA, & 0<t<\Delta \\ 0, & \text{ост. } t \end{cases}$$

5. $y(t) = A\cos(\omega_0 t + M\sin 2\pi F t)$ $\dot{a}(t)$

$\dot{a}(t) = Ae^{im\sin\Omega t}$ $\Delta F = \frac{\Omega m}{2\pi}\cos\Omega t = mF\cos\Omega t$



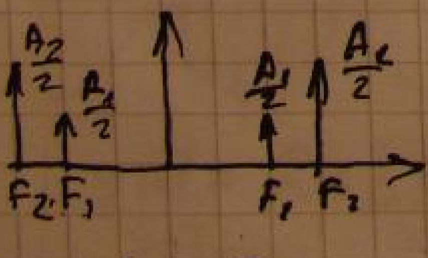


Сигнал со сложн. (неопред. мод.)

$x(t) = A_1\cos(2\pi F_1 t) + A_2\cos(2\pi F_2 t)$

$A_1 = 2$ В $F_1 = 2.2$ МГц

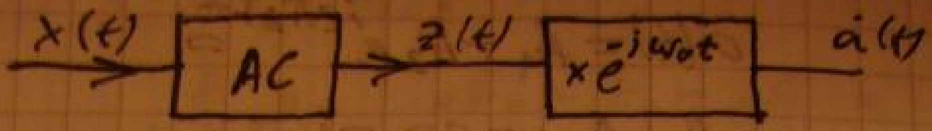
$A_2 = 3$ В $F_2 = 2.5$ МГц

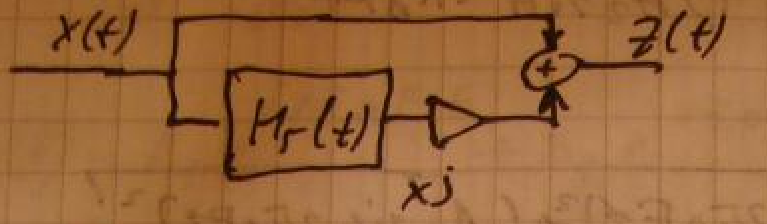


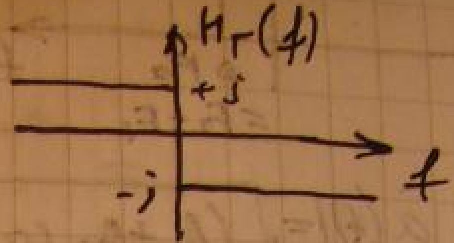
f_0 — неизвестно.

$f_0 = F_1$

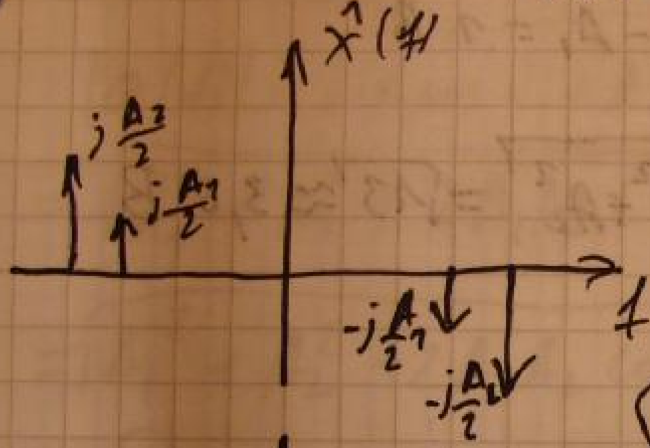
$f_0 = \cancel{F_2}\ 2{,}4$ МГц $F_1 < f_0 < F_2$



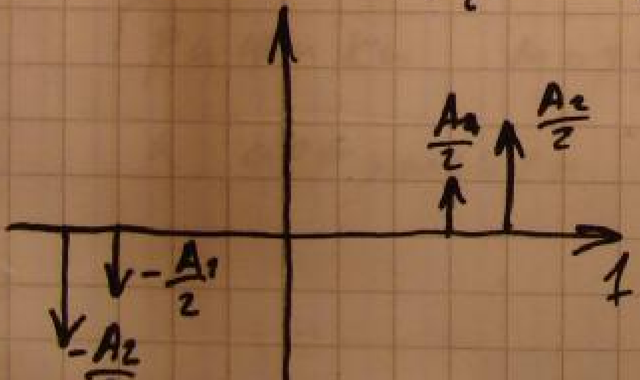




$\hat{x}(t)$ – преобр Гильбарта (сопряженный сигнал)

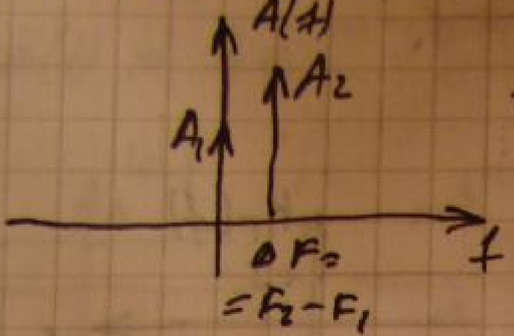


×j



$$\dot z(t) = A_1 e^{j2\pi F_1 t} + A_2 e^{j2\pi F_2 \ldots}$$

① $f_0 = F_1$



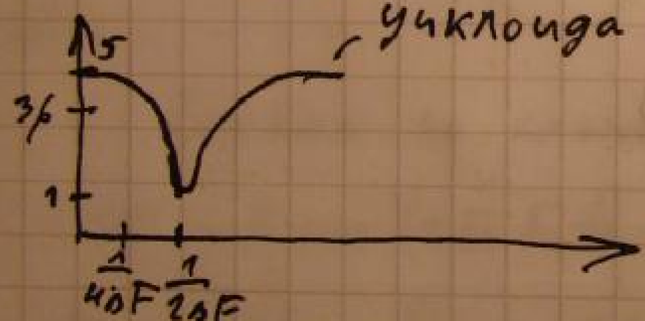
$$\dot a(t) = A_1 + A_2 e^{j2\pi\Delta F t} =$$
$$= A_1 + A_2\cos 2\pi\Delta F t +$$
$$+ jA_2 \sin 2\pi\Delta F t$$

$$|\dot a(t)| = \sqrt{(A_1 + A_2\cos 2\pi\Delta F t)^2 + (A_2 \sin 2\pi\Delta F t)^2}$$

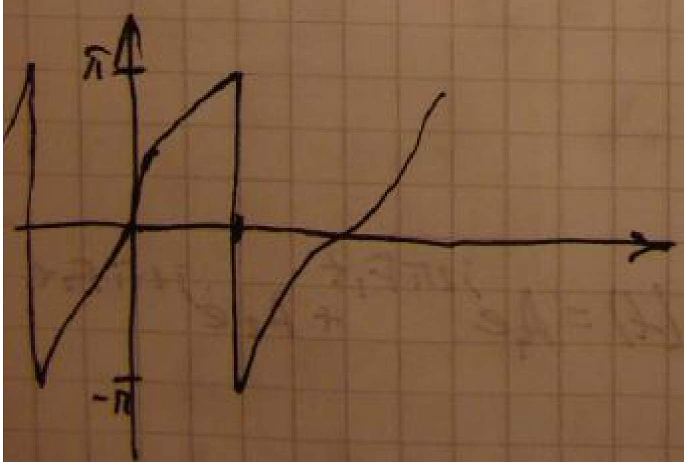
$t = 0$ $\quad |\dot a(0)| = A_1 + A_2 = 5$ В

$t = \frac{1}{2\Delta F}$ пол периода $\quad \left|\dot a\left(\frac{1}{2\Delta F}\right)\right| = A_2 - A_1 = 1$ В

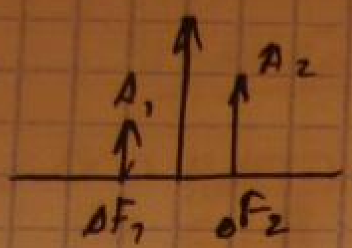
$t = \frac{1}{4\Delta F}$ четверти п. $\quad \left|\dot a\left(\frac{1}{4\Delta F}\right)\right| = \sqrt{A_1^2 + A_2^2} = \sqrt{13} \approx 3,6$ В



Arg (a(0)) = 0



② $f_0 = 2{,}4$ МГц.



$\Delta F_1 = F_1 - \Delta F = -0{,}2$ МГц

$\Delta F_2 = F_2 - \Delta F = 0{,}1$ МГц

$$\dot a(t) = A_1 e^{j2\pi \Delta F_1 t} + A_2 e^{j2\pi \Delta F_2 t} =$$

$$= A_1 \cos(\underbrace{2\pi \Delta F_1 t}_{\alpha_1}) + A_2 \cos(\underbrace{2\pi \Delta F_2 t}_{\alpha_2}) +$$

$$+ j\left(A_1 \sin(2\pi \Delta F_1 t) + A_2 \sin(2\pi \Delta F_2 t)\right)$$

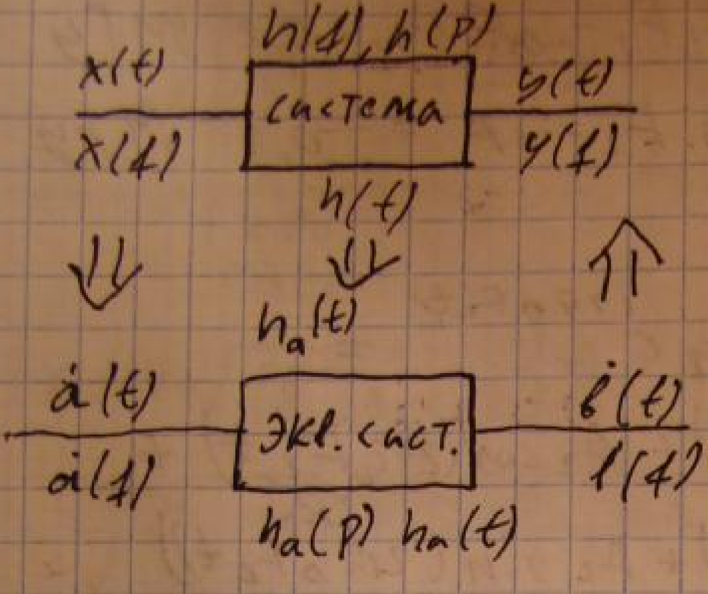
$$|a(t)| = \sqrt{(A_1\cos\alpha_1 + A_2\cos\alpha_2)^2 + (A_1\sin\alpha_1 + A_2\sin\alpha_2)^2}$$

$$= \sqrt{A_1^2 + A_2^2 + 2A_1A_2(\underbrace{\cos\alpha_1\cos\alpha_2 + \sin\alpha_1\sin\alpha_2}_{\cos(\alpha_1-\alpha_2)})} =$$

$$= \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos(2\pi \Delta F t)}$$

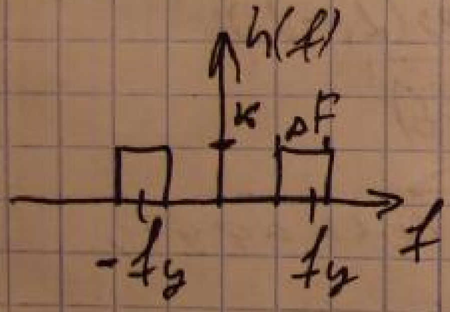
Графики модулей совпадут, а аргументы будут разные.

Лекция 20.11



⚠ $f_0 = ?$

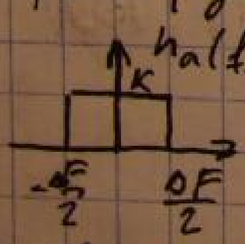
Определение ЧХ экв. цепи.



Для $f<0$ $h(f)=0$

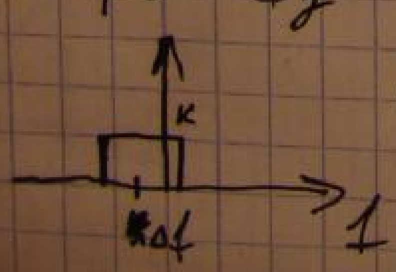
Остальное сдвинуто на f_0 влево

① $f_0 = f_y$



$$h_a(t) = K\Delta F\,\mathrm{sinc}(\pi \Delta F t)$$

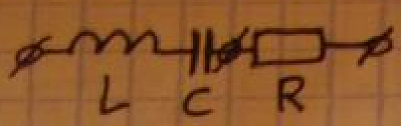
② $f_0 \neq f_y$



$$h_a(t) = K\Delta F\,\mathrm{sinc}(\pi \Delta F t)\underbrace{e^{j2\pi\Delta f t}}_{\Delta f}$$

Δf – отстройка частоты

Эквивалентная цепь для колебательного контура



$$H(p) = \frac{R}{pL + \frac{1}{pC} + R} =$$

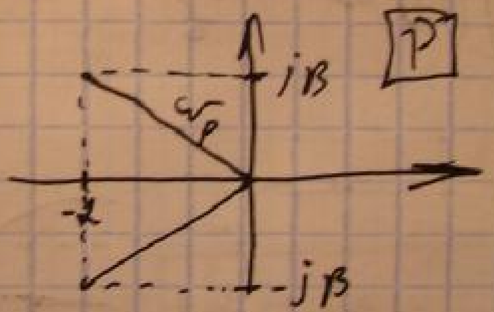
$$= \frac{pRC}{p^2 RC + pRC + 1} = \frac{\frac{R}{L} p}{p^2 + p\frac{R}{L} + \frac{1}{LC}} =$$

$$= \frac{2\alpha p}{p^2 + 2p\alpha + \omega_p^2} \qquad p_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_p^2}$$

$$p_{1,2} = -\alpha - j\sqrt{\omega_p^2 - \alpha^2} = \alpha - j\beta$$

β - частота своб. колебаний

$$\beta = \omega_p \sqrt{1 - \frac{1}{4Q^2}}$$



$$H(p) = \frac{2\alpha p}{(p - p_1)(p - p_2)} = \frac{2\alpha p}{(p + \alpha - j\beta)(p + \alpha + j\beta)} =$$

$$= \frac{c}{p + \alpha - j\beta} + \frac{c^*}{p + \alpha + j\beta}$$

$$C = \left[H_p (p + \alpha - j\beta) \right] \Big|_{-\alpha + j\beta = p} =$$

$$= \frac{2\alpha(-\alpha + j\beta)}{2j\beta} = \alpha + j\frac{\alpha^2}{\beta} = \alpha\left(1 + j\frac{\alpha}{\beta}\right) \approx$$

$$\approx \{\beta \approx \omega_p\} \approx \alpha\left(1 + j\frac{1}{2Q}\right) \approx \left\{1 \gg \frac{1}{2Q}\right\} \approx \alpha$$

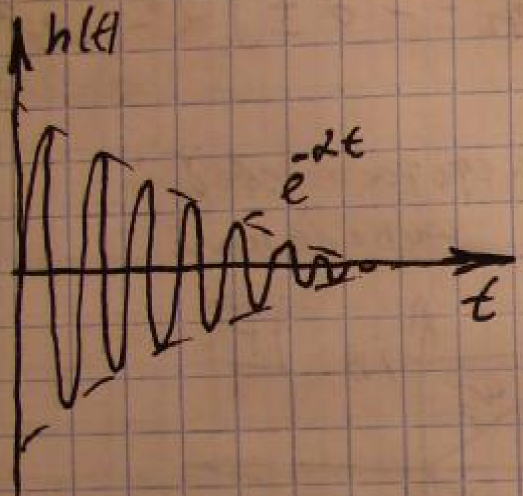
$$H(p)=\frac{\alpha}{p+\alpha-j\beta}+\frac{\alpha}{p+\alpha+j\beta}$$

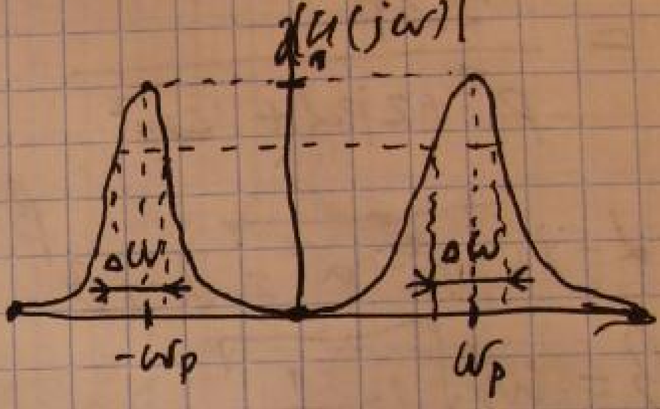
$$\boxed{e^{\gamma t}\cdot u(t) \leftrightarrow \frac{1}{p-\gamma}}$$

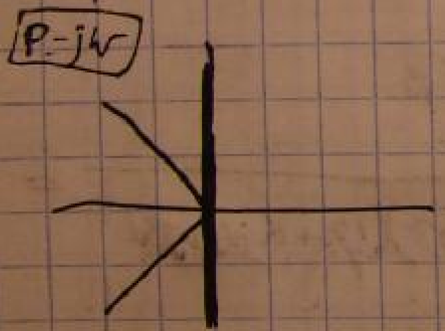
$$h(t)=\left[\alpha e^{(-\alpha+j\beta)t}+\alpha e^{(-\alpha-j\beta)t}\right]u(t)=$$

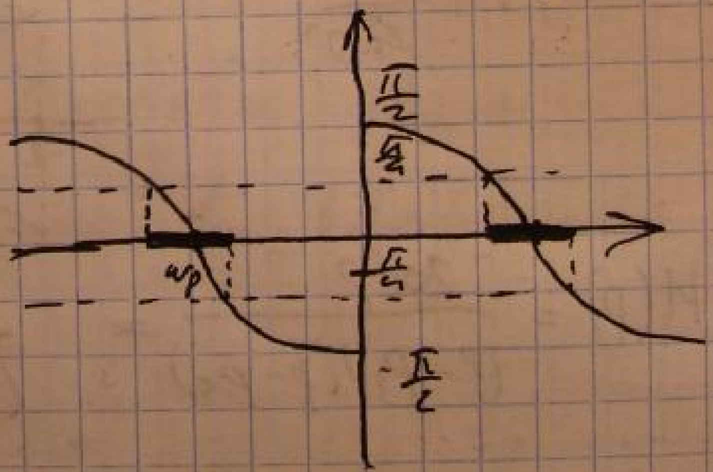
$$=2\alpha e^{-\alpha t}\cos(\beta t)\,u(t)=$$

$$=\{\beta\simeq\omega_0\}\approx 2\alpha e^{-\alpha t}\cos(\omega_0 t)\,u(t)$$







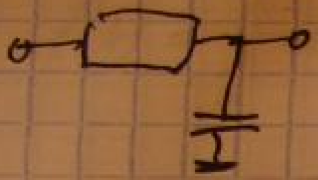


Для $\omega \gg 0$, кот. вблизи ω_p будут сост. только из $\frac{\alpha}{j\omega+\alpha-j\omega_p}$

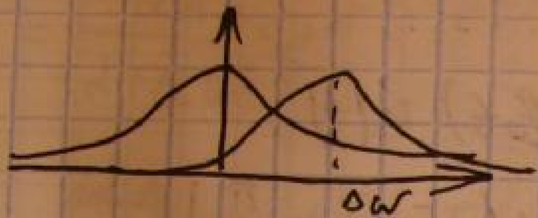
$$H(j(\omega+\omega_0))=\frac{\alpha}{\alpha+j(\omega-\Delta\omega)}=\frac{1}{1+j\frac{1}{\alpha}(\omega-\omega_0)}$$

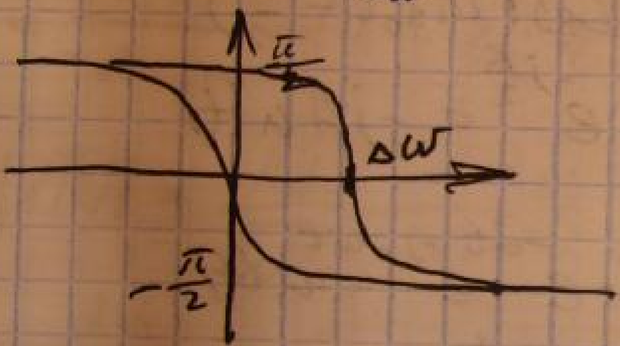
$$H_a(\omega) = \frac{1}{1+j\frac{1}{\alpha}(\omega-\Delta\omega)}$$

$$\frac{1}{1+j\frac{\omega}{\alpha}} = H_a(\omega)$$



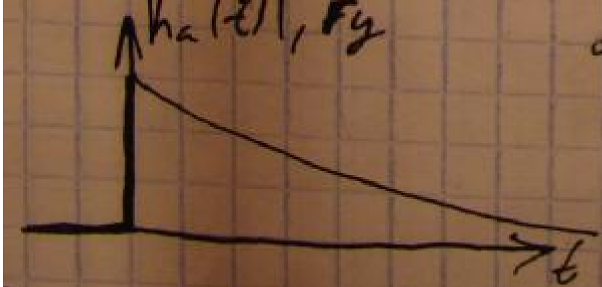
$$\frac{\frac{1}{j\omega C}}{R+j\omega L} = \frac{1}{1+j\frac{\omega}{\alpha}}$$

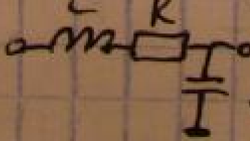




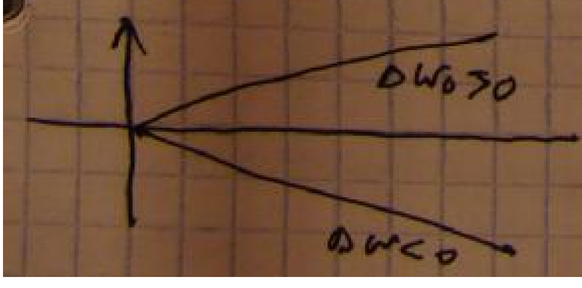
$$H_a(p) = \frac{1}{1+\frac{p}{\alpha}-j\frac{\Delta\omega}{\alpha}} = \frac{\alpha}{p+\alpha-j\Delta\omega}$$

$$h_a(t) = \alpha e^{(-\alpha+j\Delta\omega)t}u(t) = \alpha e^{-\alpha t}e^{-j\Delta\omega t}u(t)$$



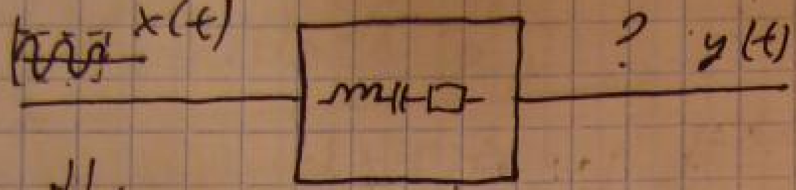


$$\Leftrightarrow H_a(\omega) = \frac{-jQ}{1+\frac{j}{\alpha}(\omega-\Delta\omega)}$$

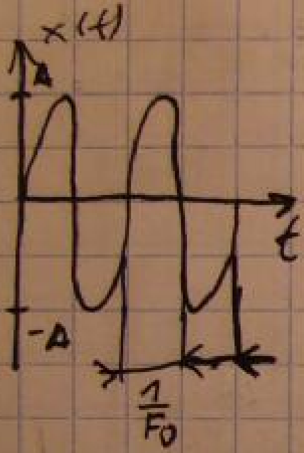


$\Leftrightarrow H_a(\omega) = ?$

Прохождение радиоимпульса через резонансный контур



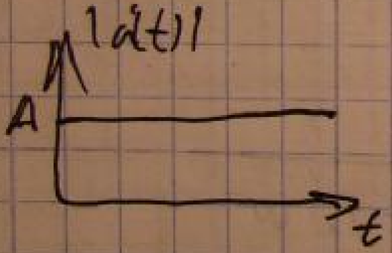
⇓ $\dot{a}(t)$ ⇓ экв. цепь



$$x(t) = A\sin(2\pi f_0 t)\, u(t)$$

$$x(t) = A\, u(t)\cdot\cos\left(2\pi f_0 t - \frac{\pi}{2}\right)$$

$$\dot{a}(t) = A\, u(t)\cdot e^{-j\frac{\pi}{2}} = -jA\, u(t)$$

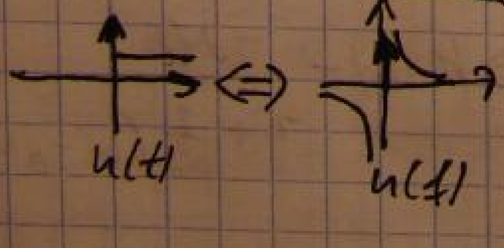


$$h_a(t) = \alpha\cdot e^{-\alpha t} e^{+j\Delta\omega t} u(t)$$

$$H_a(f) = \frac{1}{1 + j\frac{2\pi}{\alpha}(f - \Delta f)}$$

$B(f) = A(f)\cdot H_a(f)$ – частотный метод

$b(t) = a(t) * h_a(t)$ – временной метод



$$b(t) = \int_{-\infty}^{+\infty} a(\tau) \cdot h_a(t-\tau)\, d\tau =$$

$$= \int_{-\infty}^{+\infty} h_a(\tau) \cdot a(t-\tau)\, d\tau$$

$$b(t) = \int_{0}^{t} \alpha e^{(-\alpha + j\Delta\omega)\tau} \cdot (-jA)\, d\tau =$$

$$= \frac{-j\alpha A}{-\alpha + j\Delta\omega} e^{(-\alpha + j\Delta\omega)\tau} \Big|_0^t = \frac{j\alpha A}{-\alpha + j\Delta\omega} \left(e^{(-\alpha + j\Delta\omega)t} - 1 \right)$$

$$= \frac{j\alpha A}{-\alpha + j\Delta\omega} \left(1 - e^{-\alpha t} e^{j\Delta\omega t} \right)$$

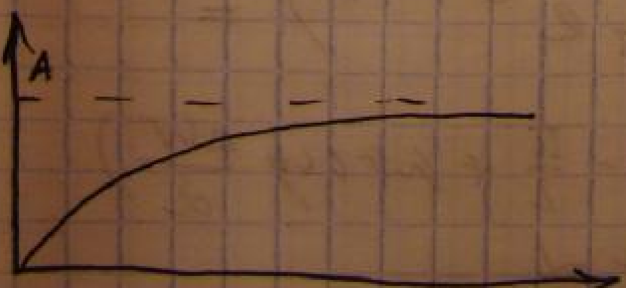
$$|b(t)| = \frac{A\alpha}{\sqrt{\alpha^2 + \Delta\omega^2}} \left| 1 - e^{-\alpha t} \cos \Delta\omega t - j e^{-\alpha t} \sin \Delta\omega t \right| =$$

$$= \frac{A\alpha}{\sqrt{\alpha^2 + \Delta\omega^2}} \sqrt{(1 - e^{-\alpha t} \cos \Delta\omega t)^2 + (e^{-\alpha t} \sin \Delta\omega t)^2} =$$

$$= \frac{A\alpha}{\sqrt{\alpha^2 + \Delta\omega^2}} \sqrt{1 - 2e^{-\alpha t} \cos \Delta\omega t + e^{-2\alpha t}}$$

$\Delta\omega = 0$

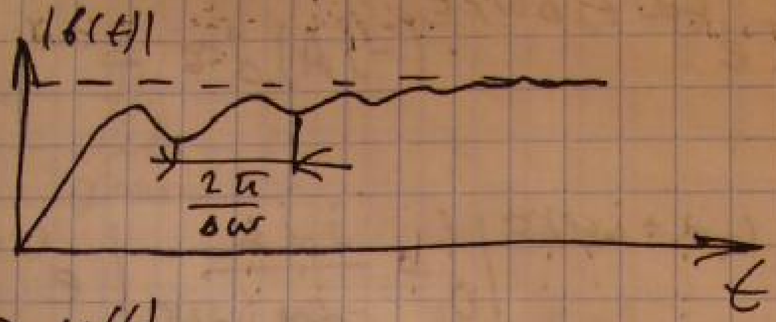
$$|b(t)| = A(1 - e^{-\alpha t})\, u(t)$$



$\Delta\omega \neq 0$

$$|b(t)| = \frac{A\alpha}{\sqrt{\alpha^2 + \Delta\omega^2}} \sqrt{1 - 2e^{-\alpha t}\cos\Delta\omega t + e^{-2\alpha t}}$$

$$t \to \infty \qquad |b(t)| \to \frac{A\alpha}{\sqrt{\alpha^2 + \Delta\omega^2}} < A$$



$b(t) \to y(t)$

$$y(t) = Re\left\{ b(t) \cdot e^{j\omega_0 t} \right\}$$

$$y(t) = Re\left\{ \frac{jA\alpha}{-\alpha + j\Delta\omega}\left(1 - e^{-\alpha t} e^{-j\Delta\omega t}\right) e^{j\omega_0 t} \right.$$

$$y(t) = y_1(t) - y_0(t)$$

$$y_1(t) = Re\left\{ \frac{jA\alpha}{-\alpha + j\Delta\omega} e^{j\omega_0 t} \right.$$

$$\left| \frac{jA\alpha}{\underbrace{-\alpha + j\Delta\omega}_{*}} \right| \qquad \frac{A\alpha}{\alpha^2 - \Delta\omega^2}$$

$$\arg(*) = \frac{\pi}{2} - \left(\pi + \mathrm{arctg}\,\frac{\Delta\omega}{-\alpha}\right) = \left[-\frac{\pi}{2} + \mathrm{arctg}\,\frac{\Delta\omega}{\alpha}\right.$$

$$y_1(t) = Re\left(\frac{A\alpha}{\sqrt{\alpha^2 + \Delta\omega^2}} e^{j(\omega_0 t + \varphi)} \right) =$$

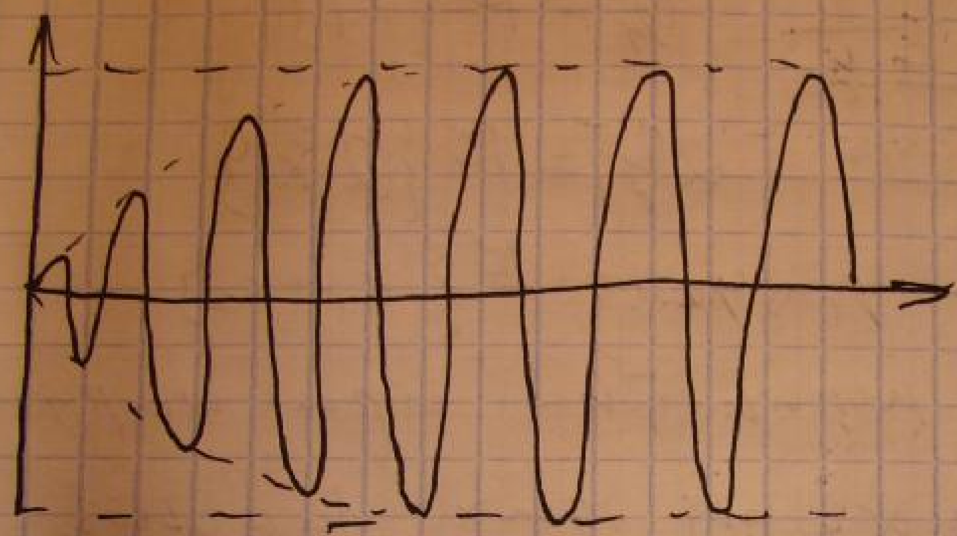
$$= \frac{A\alpha}{\sqrt{\alpha^2 + \Delta\omega^2}} \cos\left(\omega_0 t - \frac{\pi}{2} + \mathrm{arctg}\,\frac{\Delta\omega}{\alpha}\right)$$

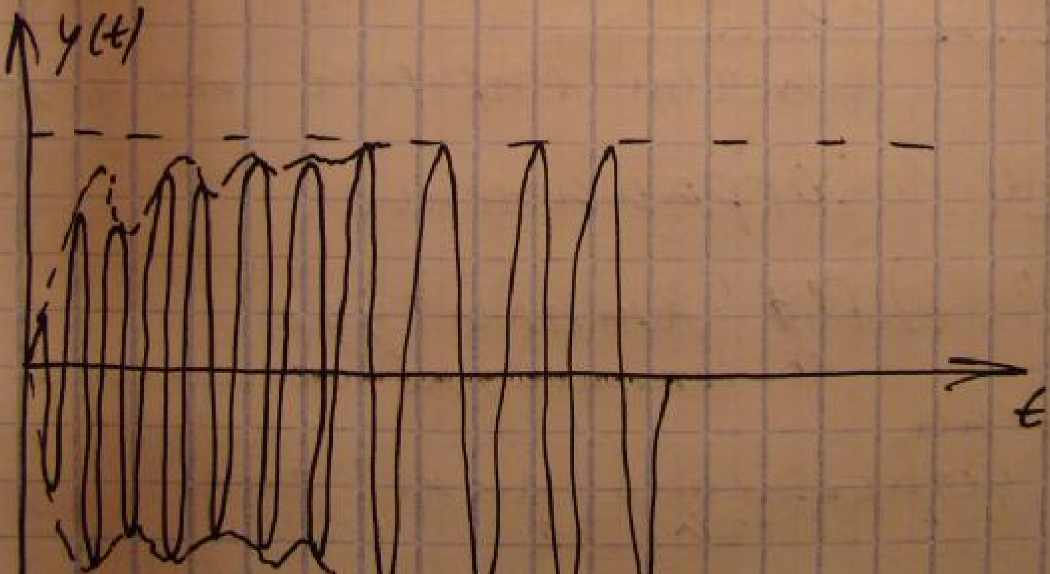
⚠ $\Delta\omega \sim \alpha$

$$y_2(t) = \mathrm{Re}\left\{ \frac{jA\alpha}{-\alpha + j\omega_0} e^{-\alpha t} \cdot e^{j\omega_p t} \right.$$

$$\Delta\omega = \omega_p - \omega_0$$

$$y_2(t) = \frac{A\alpha}{\sqrt{\alpha^2 + \Delta\omega^2}} e^{-\alpha t} \cos\left(\omega_p t - \frac{\pi}{2} + \mathrm{arctg}\, \frac{\Delta\omega}{\alpha}\right)$$

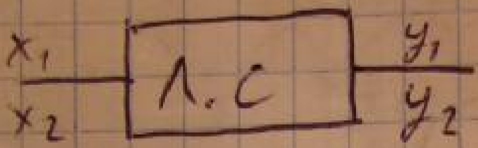




Лекция 04.12

Нелинейные устройства

Введение в теорию нелинейных устройств

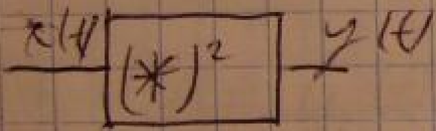


$$y_1 = F\{x_1\}$$

$$\alpha y_1 = F\{\alpha x_1\}$$

$$y_1 + y_2 = F\{x_1 + x_2\}$$

Система нелинейна при нарушении хотябы одного из этих законов.



Абсолютно линейных систем в природе не сущ.

Ограничения

·) По амплитуде
не большой, не маленький

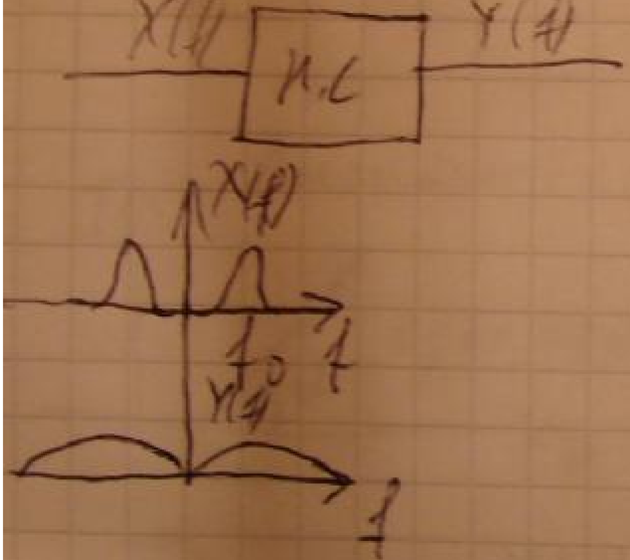
·) По времени.
время установления лин. раб.

$$I = C\frac{du}{dt}$$

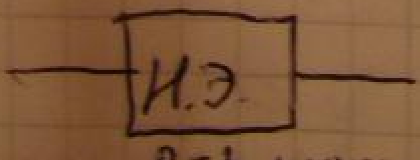
При $U > 10$ МВ - пробой

Частотные признаки линейности

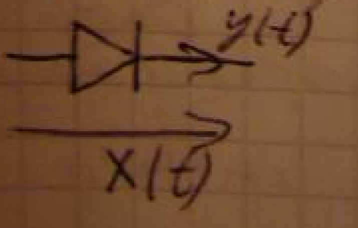
Y(f) - может быть ненулевым, только на тех частотах где X(f) ненулевой (Л.С. не порождает дополнит. спектральных составляющих)



Нелинейные элементы (НЭ)

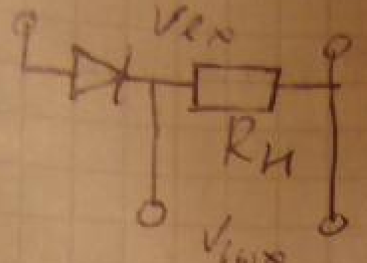


p-n переход



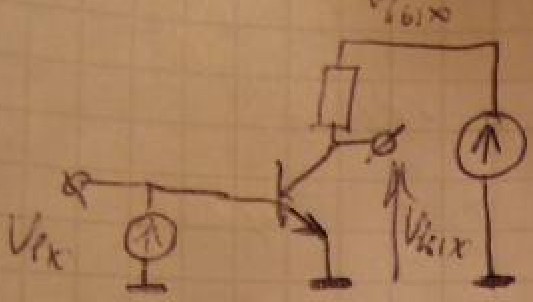
x(t) v(t)

y(t) - i(t)



$R_н \ll R_{рп}$

$V_{вых} \ll V_{вх}$



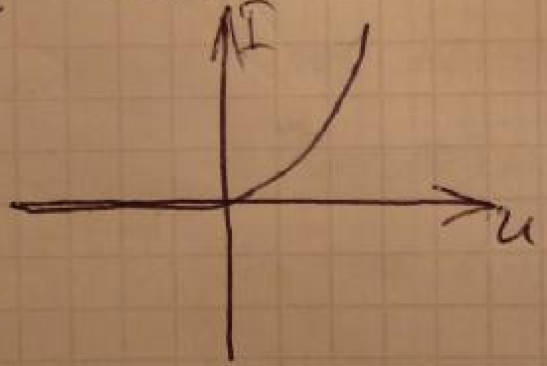
$V_{вых} \sim V_{вх}$

$V_{вых} \gg V_{вх}$

Описание НЭ

$$I = I_0\left[\exp\left(\frac{V - V_s}{\varphi} - 1\right)\right]$$

$\varphi_{кремн.} = 25$ мВ



Аппроксимация ВАХ

$I = f(V)$

Рабочая точка ВАХ

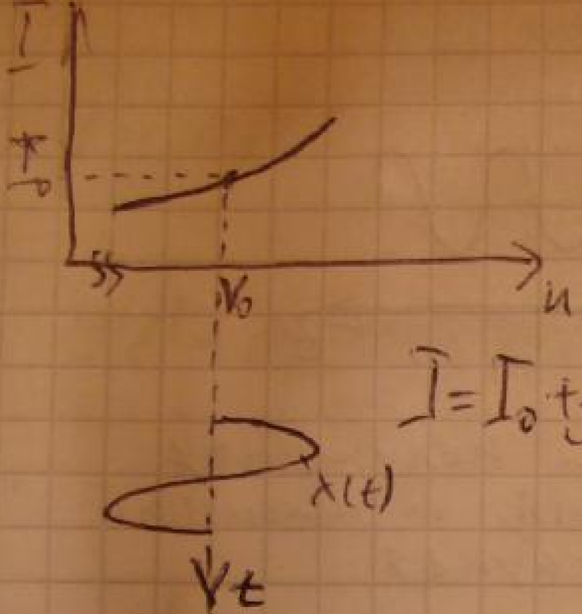
Выбор раб. точки опр. амплитуда сигнала

Она же опр. способ аппроксимации

$|v(t)| \leq \varphi$ — малый сигнал.

$|v(t)| \geq \varphi$ — большой сигнал

Аппроксимация в режиме малого сигнала (Полиномиальная)



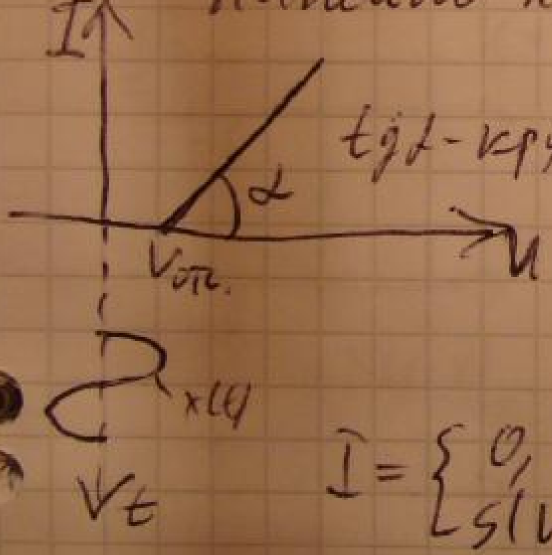
$$V(t) = V_0 + x(t)$$

$$I = f(V)$$

$$I_0 + A_1(V - V_0) + A_2(V - V_0)^2 + A_3(V - V_0)^3$$

$$I = I_0 + \underbrace{A_1 x + A_2 x^2 + A_3 x^3}_{y}$$

Линейно-ломаная (или кусочная) аппроксимация



$tg\,\alpha$ – крутизна S

$$I = f(V) = \begin{cases} S(V - V_{отс}) & V \gg V_{отс} \\ 0, & ост.\ V \end{cases}$$

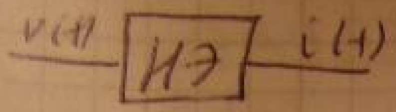
$$I = \begin{cases} 0, & V_0 < V_{отс} \\ S(V - V_0) & V_0 > V_{отс} \end{cases}$$

$$V(t) \gg \varphi$$

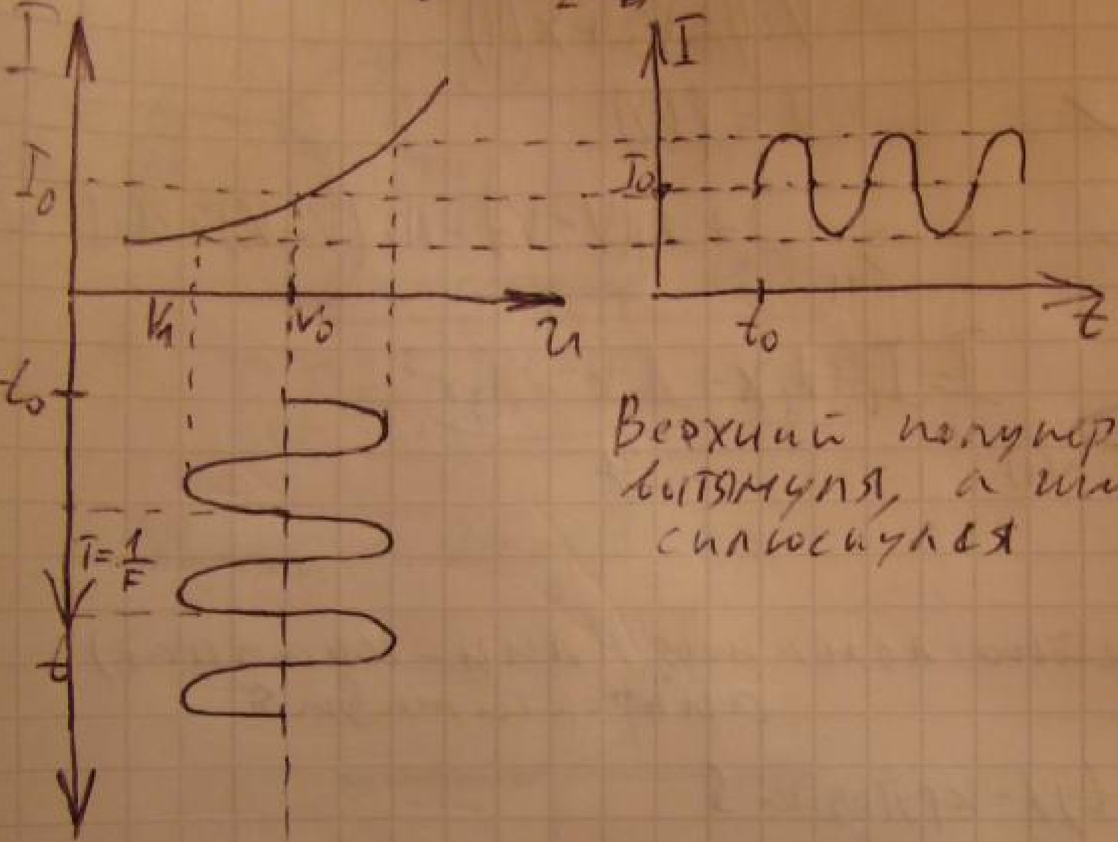
$$V > 10\,В$$

$$S = \frac{1}{R_{вн}}$$

Квадратичный НЭ



$$I = a_0 + a_1 V + a_2 V^2$$



Верхний полупериод вытянулся, а нижний сплюснулся

$$V(t) = V_0 + x(t)$$

$$x(t) = V_1 \cos(\omega t)$$

$$I = I_0 + A_1(V - V_0) + A_2(V - V_0)^2$$

$$V(t) = V_0 + V_1 \cos \Omega t$$

$$I = I_0 + A_1 V_1 \cos \Omega t + A_2 V_1^2 \cos^2 \Omega t$$

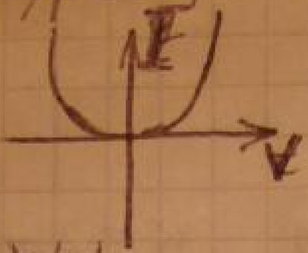
$$I = I_0 + \underbrace{A_1 V_1 \cos \Omega t + A_2 V_1^2/2 + \frac{A_2 V_1^2}{2} \cos 2\Omega t}_{y(t)}$$

$$y(t) = \frac{a_2 V_1^2}{2} + a_1 V_1 \cos \Omega t + \frac{a_2 V_1^2}{2} \cos 2\Omega t$$

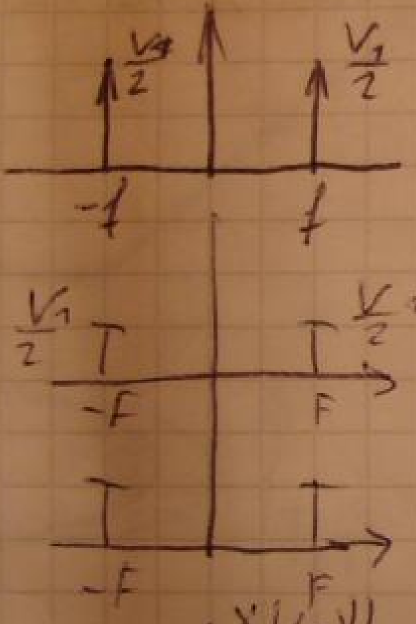
$$i(t) = \left[I_0 + \frac{a_2 V_1^2}{2}\right] + a_1 V_1 \cos \Omega t + \frac{a_2 V_1}{2} \cos 2\Omega t$$

Квадратор

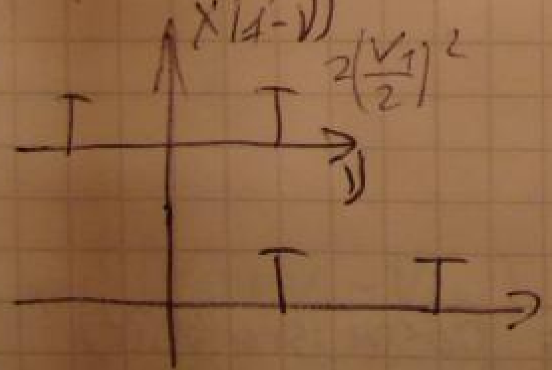
$$y = a_2 V^2$$



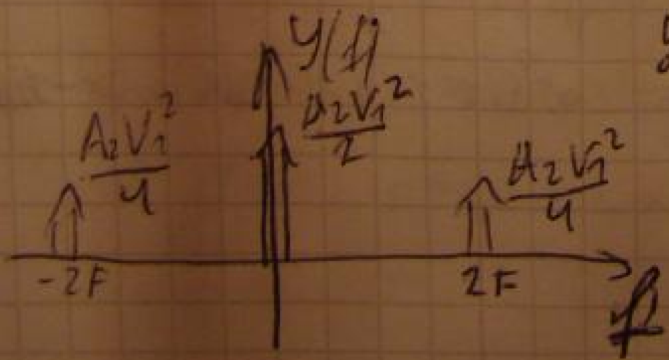
$$Y(f) = a_2\, X(f) * X(f)$$



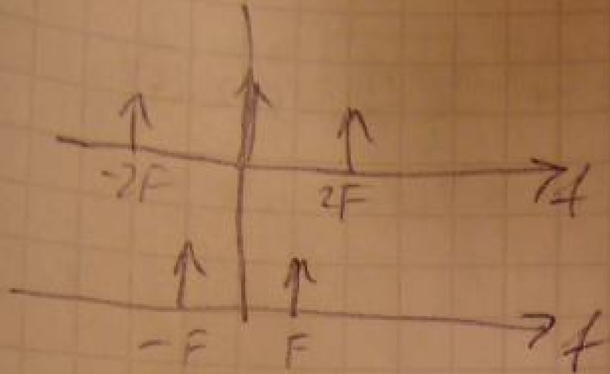
$$X(f) * X(f) = \int_{-\infty}^{+\infty} X(\nu) \cdot X(f-\nu)\, d\nu$$



$$y(t) = \frac{a_2 V_1^2}{2} + \frac{a_2 V_1^2}{2} \cos(2\pi\, 2F t)$$



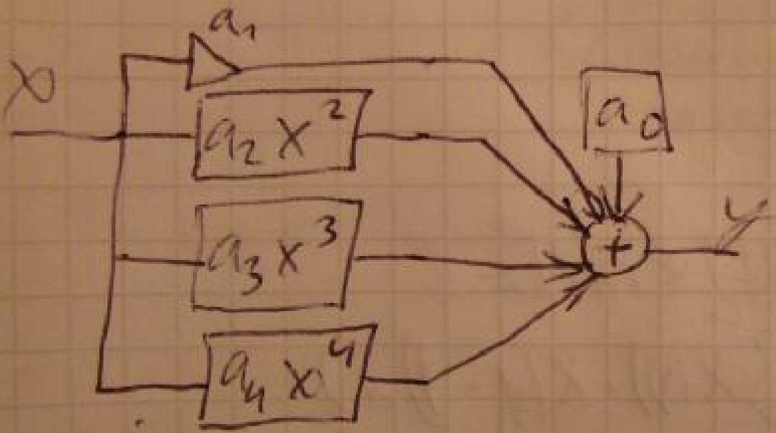
$$Y(f) = a_3 \cdot \underbrace{Y(f) * Y(f) * Y(f)}_{\text{триспектр}}$$



$$y = x^n \qquad F = k \cdot F$$

$k \le n$ $\quad$ $k = 2n+1$ если n нечётн.

$k = 2n$ если n чётн.



$$e^{jx} = \cos x + j \sin x$$

$$\cos \Omega t \Rightarrow e^{j\Omega t}$$

$$\cos N\Omega t \Rightarrow e^{nj\Omega t}$$

$$\left(e^{j\Omega t}\right)^n = e^{jn\Omega t}$$

$$\cos N\Omega t + j \sin N\Omega t = (\cos \Omega t + j \sin \Omega t)^n$$

$N = 2$

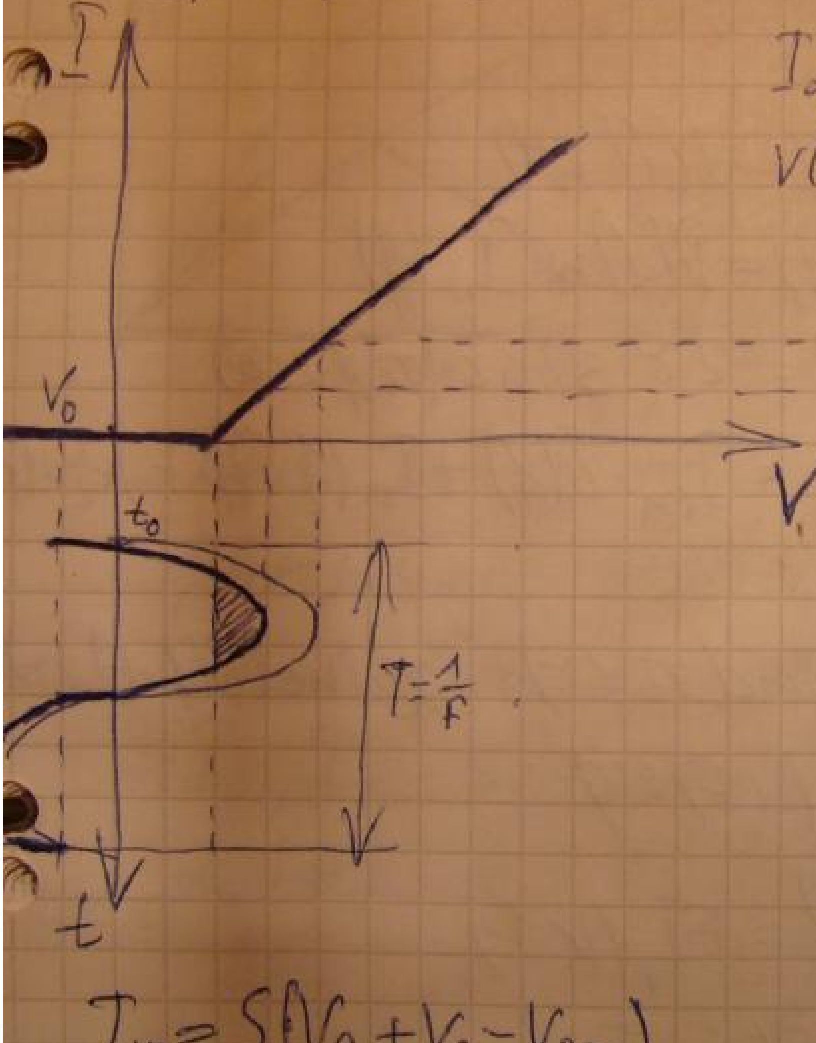
$$\cos 2\Omega t + j \sin 2\Omega t = \cos^2 \Omega t + 2j \sin \Omega t \cos \Omega t - \sin^2 \Omega t$$

Лекция 11.12

Кусочно-линейная аппроксимация ВАХ

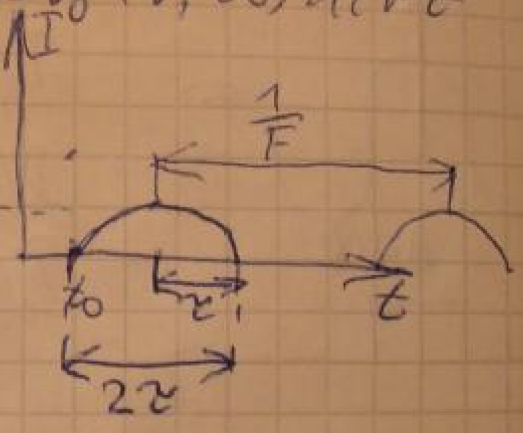
$$I = \begin{cases} S(V - V_{отс}), & V > V_{отс} \\ 0, & V \le V_{отс} \end{cases}$$

$v(t) = V_1 \cos(2\pi F t)$



I_0, V_0 – раб. точка

$v(t) = V_0 + V_1 \cos 2\pi F t$



$I_m = S(V_0 + V_1 - V_{отс})$

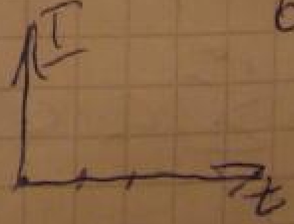
Имп. хар 3 парам:

•) Период, частота

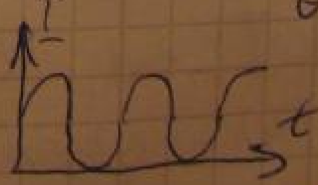
•) Угол отсечки θ

$\frac{\theta}{2\pi} = \frac{\tau}{T}$ $\quad 0 \le \theta \le \pi$

$\theta = 0$



$\theta = 180$

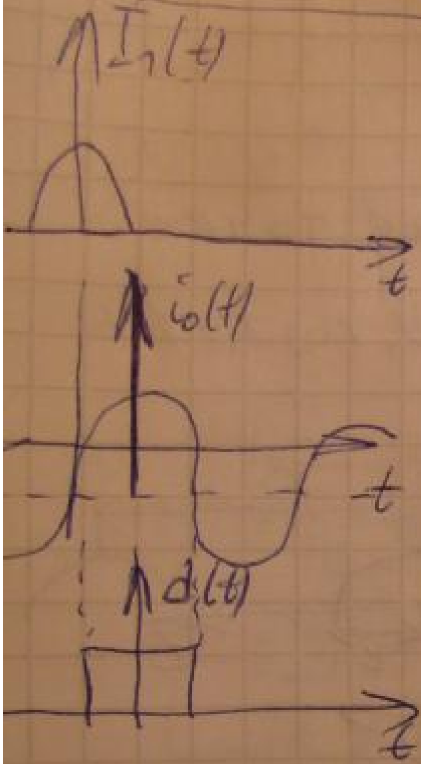


Для произв угла отсечки!

$V_0 + V_1 \cos 2\pi F\tau = V_{отс}$.

$\cos 2\pi F\tau = \frac{V_{отс} - V_0}{V_1}$

Частотный анализ



$I_1(f) = ?$

$i_1(t) = d(t)\, i_0(t)$

$I_1(f) = D(f) * I_0(f)$

$D(f) = 2\tau\, \mathrm{sinc}(\pi f 2\tau)$

$I_0(f) = I_{см} \cdot \delta(f) + I_a\left(\frac{1}{2}\delta(f-F) + \frac{1}{2}\delta(f+\right.$

$I_1(f) = I_{см} \cdot 2\tau\, \mathrm{sinc}(2\pi f \tau) + I_a\tau\, \mathrm{sinc}(\pi(f-F)2\tau) +$
$+ I_a\tau\, \mathrm{sinc}(\pi(f+F)2\tau)$

$c_k = G(k) = \frac{I_1(\frac{k}{T})}{T} = F I_1(kF)$

$i(t) = \sum_{k=-\infty}^{+\infty} c_k \cdot e^{j2\pi kFt} = \sum_{k=0}^{+\infty} I_k \cos(2\pi kFt)$

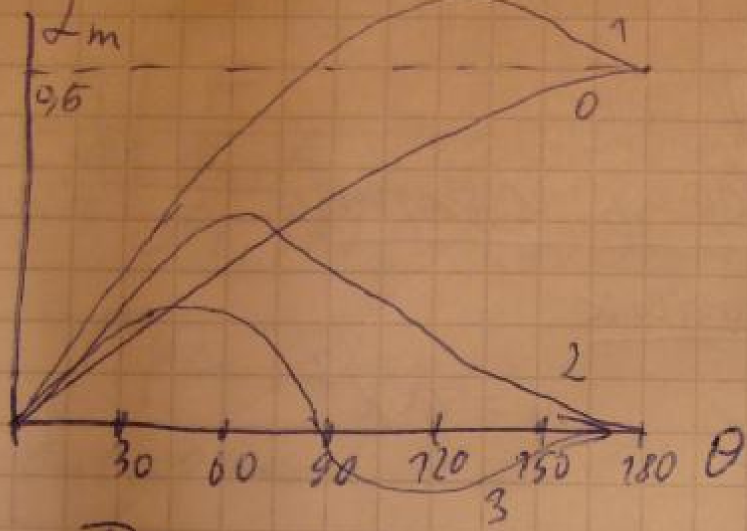
$I_k = I_m \cdot \alpha_k(\theta)$

$\alpha_k(\theta)$ – функция Берга

$$\alpha_0(\theta) = \frac{\sin\theta - \theta\cos\theta}{\pi(1-\cos\theta)}$$

$$\alpha_1(\theta) \quad \frac{\theta - \sin\theta\cos\theta}{\pi(1-\cos\theta)}$$

$$\alpha_{n\geq 2}(\theta) = \frac{2(\sin(n\theta)\cos\theta - n\sin\theta\cos(n\theta))}{\pi(1-\cos\theta)n(n^2-1)}$$



При мал. угл. отсечки функции Берга соизмеримы.
При бол. θ явно присутств. I_0, I_1 остальные малы.
$\alpha_n(\theta)$ $\theta \to \alpha_n = max$

При подач. гарм. на НЭ ВАХ кот. апроксимировано линейно-ломаной фу-ей, на выходе (ток через элемент) получ. периодич. посл. импульсов: Период равен периоду гармоники

Ампл. равна $I_m =$

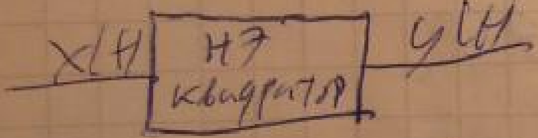
Угол отсечки равен: $\theta =$

Спектр вых сигнала состоит из гармоник с частотой nF

$$I_n = I_m \cdot \alpha_k(\theta)$$

НЭ преобраз суммы гармоник

$$y = ax^2$$



Пусть $x(t) = A_1\cos(\omega_1 t) + A_2\cos(\omega_2 t)$

$$\omega = 2\pi F t$$

$$y(t) = a(A_1\cos(\omega_1 t) + A_2\cos(\omega_2 t))^2 =$$

$$= a(A_1^2\cos^2(\omega_1 t) + 2A_1A_2\cos\omega_1 t\cos\omega_2 t + A_2^2\cos^2(\omega_2 t))$$

$$\boxed{\begin{aligned}\cos^2\alpha &= \frac{1}{2} + \frac{1}{2}\cos 2\alpha \\ \cos\alpha\cos\beta &= \frac{1}{2}[\cos(\alpha+\beta) + \cos(\alpha-\beta)]\end{aligned}}$$

$$= a[\frac{A_1^2}{2} + \frac{A_1^2}{2}\cos 2\omega_1 t + \frac{A_2^2}{2} + \frac{A_2^2}{2}\cos 2\omega_2 t +$$

$$+ A_1A_2\cos(\omega_1+\omega_2)t + A_1A_2\cos(\omega_1-\omega_2)t$$

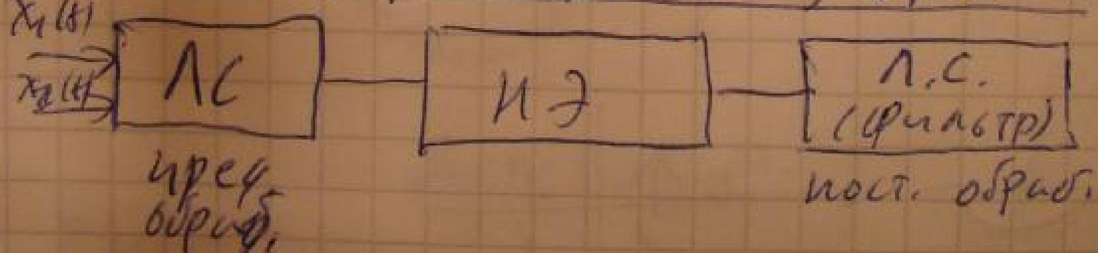
n	F
0	0
2	$2F_1, 2F_2, F_1-F_2, F_1+F_2$

$N=3$

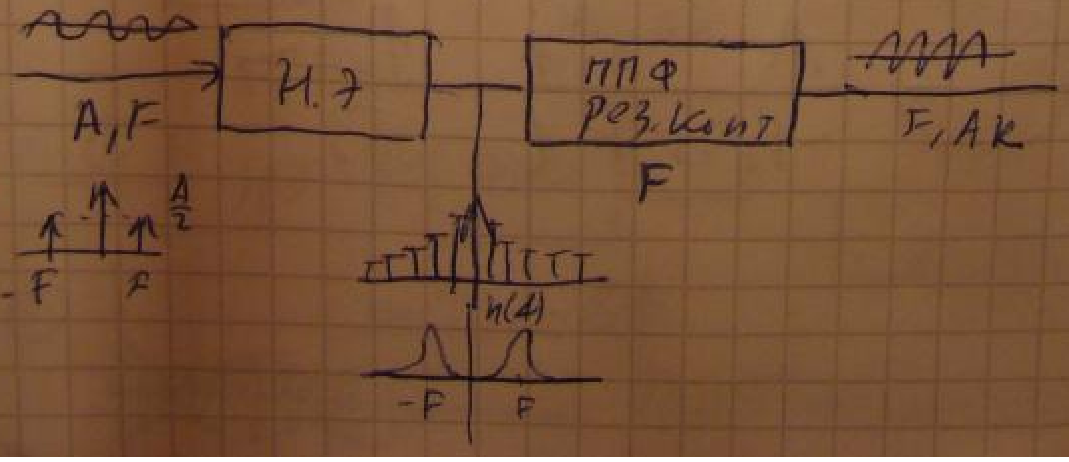
n	F
1	F_1, F_2
3	$3F_1, 3F_2;\ \lvert F_1 \pm 2F_2 \rvert;\ \lvert 2F_1 \pm F_2 \rvert$

При подаче на НЭ бигармонического сигнала на вых образ кратные и комбинационные гармоники.

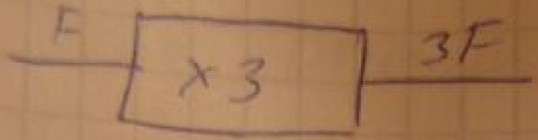
Примеры НЛ устройств

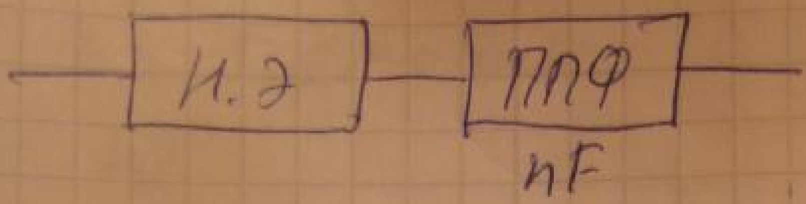


НЛ усилитель

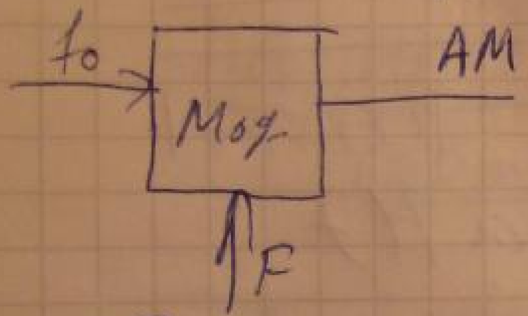


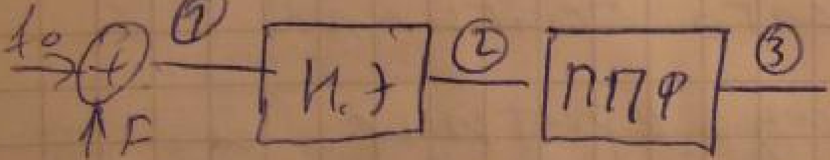
Умножители частоты

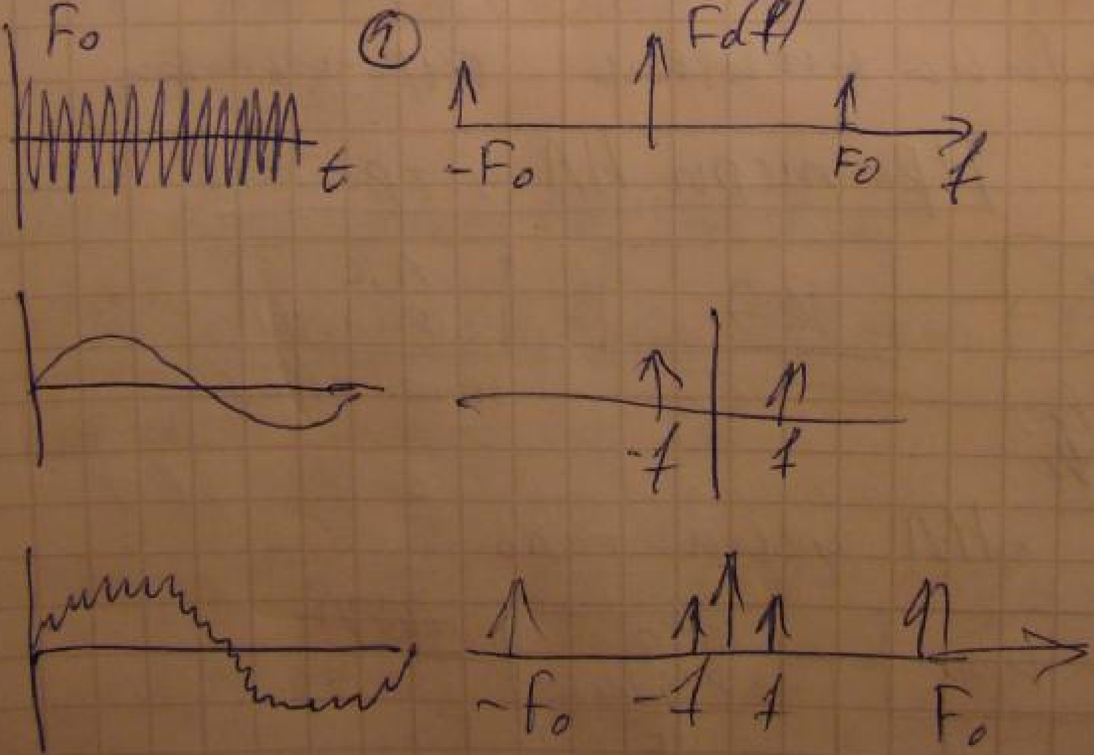


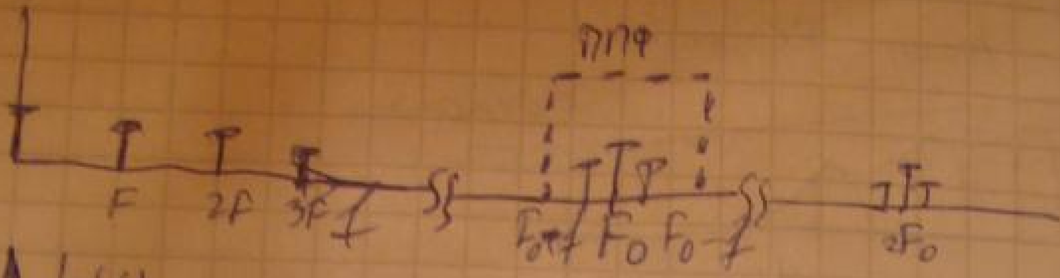


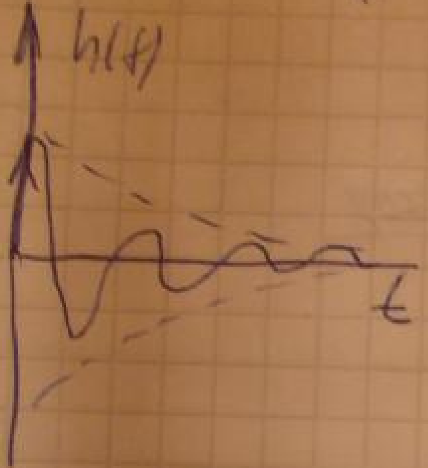
Модуляторы

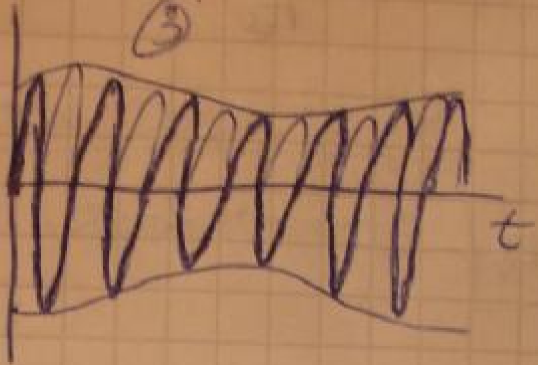




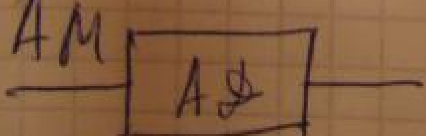




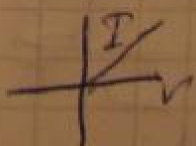


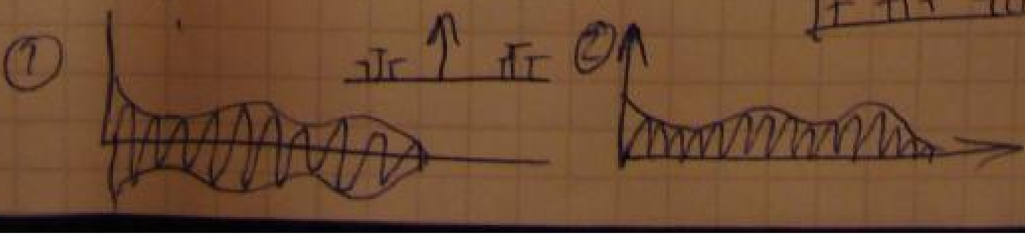


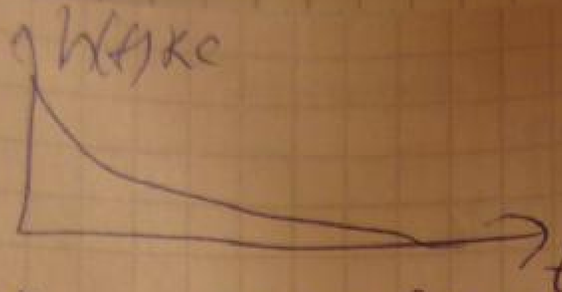
Детекторы

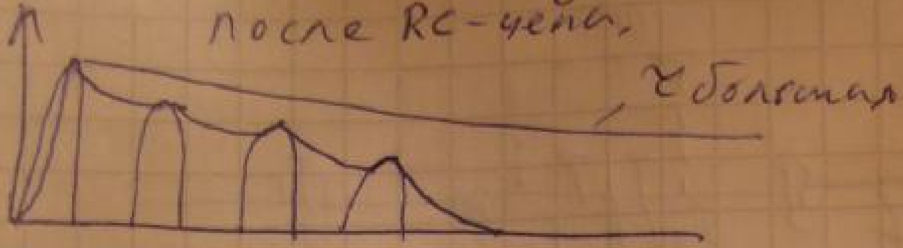


Закон
для
аппр.



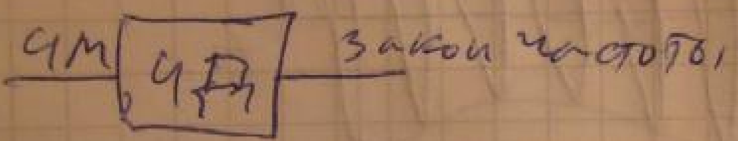


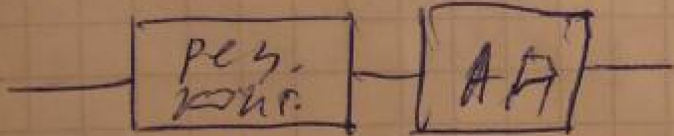




$$\frac{1}{F_0} << \tau << \frac{1}{F_B}$$

Частотный детектор





$F_p \neq F_0$

$f_{0,н} = F_0$